Aggregate Demand and the Dynamics of Unemployment

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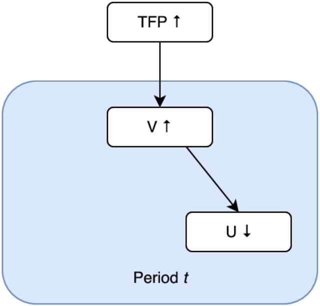
- Benchmark model of equilibrium unemployment features too little amplification and propagation of shocks
- Revisit traditional view that depressed aggregate demand can lead to persistent unemployment crises
- We augment the **DMP** model with monopolistic competition a la **Dixit-Stiglitz**
  - High aggregate demand leads to more vacancy posting
  - More vacancies lower unemployment and increase demand

Mechanism generates *amplification* and *propagation* of shocks:



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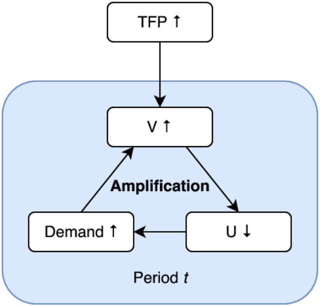

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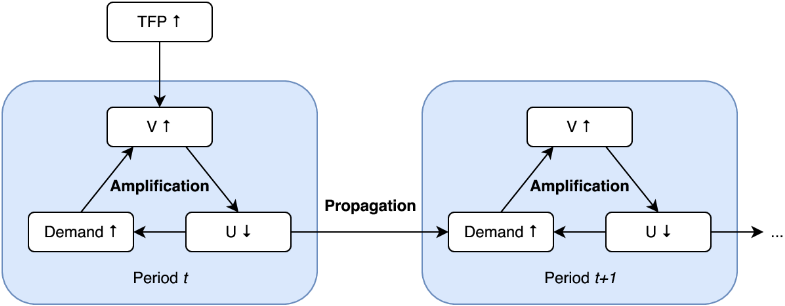

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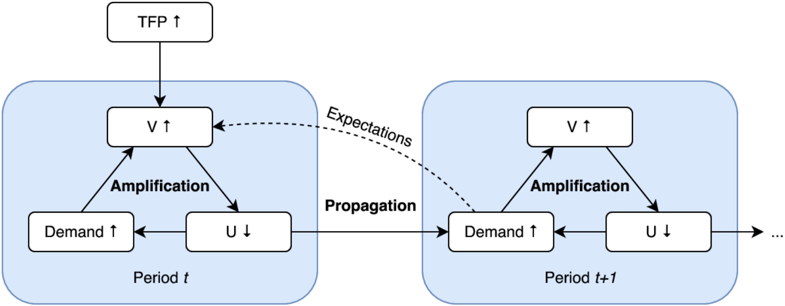

Introduction

- Aggregate demand channel adds a positive feedback loop
  - Multiple equilibria naturally arise
    - Issues with quantitative/policy analysis
    - Multiplicity sensitive to hypothesis of homogeneity
  - Introducing heterogeneity leads to uniqueness
    - Study coordination issues without indeterminacy
- Unique equilibrium with heterogeneity features interesting dynamics
  - Non-linear response to shocks
  - Multiple steady states, possibility of large unemployment crises

Literature

- NK models with unemployment
  - Blanchard and Gali, 2007; Gertler and Trigari, 2009; Christiano et al., 2015
  - Linearization removes effects and ignores multiplicity
- Multiplicity in macro
  - Cooper and John (1988), Benhabib and Farmer (1994)...
  - Search models: Diamond (1982), Diamond and Fudenberg (1989), Howitt and McAfee (1992), Mortensen (1999), Farmer (2012), Sniekers (2014), Kaplan and Menzio (2015), Eeckhout and Lindenlaub (2015), Golosov and Menzio (2016)
- Dynamic games of coordination
  - Chamley (1998), Angeletos, Hellwig and Pavan (2007), Schaal and Taschereau-Dumouchel (2015)
- Unemployment-volatility puzzle
  - Shimer (2005), Hagedorn and Manovskii (2008), Hall and Milgrom (2008)
- Multiple steady states in U.S. unemployment data
  - Sterk (2016)

I. Model

Model

- Infinite horizon economy in discrete time
- Mass 1 of risk-neutral workers
    - Constant fraction s is self-employed
    - Fraction $1 - s$ must match with a firm to produce
    - Denote by u the mass of unemployed workers
    - Value of leisure of b

Model

- Final good used for consumption
- Unit mass of differentiated goods j used to produce the final good
  - Good j is produced by worker j
  - Output

$$Y_j = \begin{cases} Ae^z & \text{if worker } j \text{ is self-employed or matched with a firm} \\ 0 & \text{otherwise} \end{cases}$$

where $A > 0$ and $z' = \rho z + \varepsilon_z$.

Final good producer

- The final good sector produces

$$Y = \left(\int_0^1 Y_j^{\frac{\sigma - 1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma - 1}}, \; \sigma > 1$$

  yielding demand curve

$$Y_j = \left(\frac{P_j}{P} \right)^{-\sigma} Y$$

  and we normalize $P = 1$.

- Revenue from production

$$P_j Y_j = Y^{\frac{1}{\sigma}} \left(Ae^z \right)^{1 - \frac{1}{\sigma}} = (1 - u)^{\frac{1}{\sigma - 1}} Ae^z$$

Nb firms

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Nb firms

Labor Market

- With v vacancies posted and u workers searching, define $\theta \equiv v/u$
  - A vacancy finds a worker with probability $q(\theta)$
  - A worker finds a vacancy with probability $p(\theta) = \theta q(\theta)$
- Jobs are destroyed exogenously with probability $\delta > 0$

Timing

Timing

1. u workers are unemployed, productivity z is drawn
2. Production takes place and wages are paid
3. Firms post vacancies and matches are formed. Incumbent jobs are destroyed with probability δ.

Unemployment follows

$$u' = (1 - p(\theta))\, u + \delta\, (1 - s - u)$$

Problem of a Firm

Value functions
Value of a firm with a worker is

$$J(z, \quad) = P_j Y_j - w + \beta (1 - \delta) E \left[J \left(z', \quad \right) | z \right].$$

The value of an employed worker is

$$W(z, \quad) = w + \beta E \left[(1 - \delta) W \left(z', \quad \right) + \delta U \left(z', \quad \right) \right],$$

and the value of an unemployed worker is

$$U(z, \quad) = b + \beta E \left[p(\theta) W \left(z', \quad \right) + (1 - p(\theta)) U \left(z', \quad \right) \right].$$

Nash bargaining

$$w = \gamma P_j Y_j + (1 - \gamma) b + \gamma \beta p(\theta) E \left[J \left(z', \quad \right) \right]$$

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Entry Problem

- Each period, a large mass M of firms can post a vacancy at a cost of $\kappa \sim$ iid $F(\kappa)$ with support $[\underline{\kappa}, \overline{\kappa}]$ and dispersion σ_κ
- A potential entrant posts a vacancy iif

$$q(\theta)\,\beta E\left[J\left(z', u'\right)\right] \geqslant \kappa.$$

- There exists a threshold $\hat{\kappa}(z, u)$ such that firms with costs $\kappa \leqslant \hat{\kappa}(z, u)$ post vacancies

$$\hat{\kappa}(z, u) = \begin{cases} \overline{\kappa} & \text{if } \beta q\left(\frac{M}{u}\right) E\left[J\left(z', u'\right)\right] > \overline{\kappa} \\ \kappa \in [\underline{\kappa}, \overline{\kappa}] & \text{if } \beta q\left(\frac{MF(\kappa)}{u}\right) E\left[J\left(z', u'\right)\right] = \kappa \\ \underline{\kappa} & \text{if } \beta q(0)\, E\left[J\left(z', u'\right)\right] < \underline{\kappa} \end{cases}$$

Note: there can be multiple solutions to the entry problem.

Equilibrium Definition

Definition

A recursive equilibrium is a set of value functions for firms $J(z,u)$, for workers $W(z,u)$ and $U(z,u)$, a cutoff rule $\hat{\kappa}(z,u)$ and an equilibrium labor market tightness $\theta(z,u)$ such that

1. The value functions satisfy the Bellman equations of the firms and the workers under the Nash bargaining equation
2. The cutoff $\hat{\kappa}$ solves the entry problem
3. The labor market tightness is such that $\theta(z,u) = MF(\hat{\kappa}(z,u))/u$, and
4. Unemployment follows its law of motion

II. Multiplicity and Non-linearity

Equilibrium Characterization

- Define the expected benefit of entry for the marginal firm $\hat{\kappa}$

$$\Psi(z, u, \hat{\kappa}) \equiv q(\theta(\hat{\kappa}))\, \beta E\left[J\left(z', u'(\hat{\kappa})\right)\right] - \hat{\kappa}$$

  - At an interior equilibrium, $\Psi = 0$

Equilibrium Characterization

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Equilibrium Characterization

$$\Psi(z,u,\hat{\kappa}) \equiv \underbrace{q(\theta(\hat{\kappa}))}_{(1)} \beta E\left[J\left(z', \underbrace{u'(\hat{\kappa})}_{(2)}\right)\right] - \underbrace{\hat{\kappa}}_{(3)}$$

Forces at work

(1) Crowding out: more entrants lower probability of match

(2) Demand channel: more entrants increase demand

(3) Cost: more entrants increase marginal cost κ

Number of equilibria

- (1) and (3) are substitutabilities → unique equilibrium
- (2) is a complementarity → multiple equilibria

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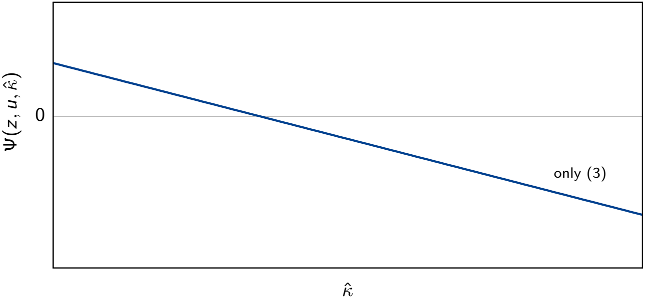
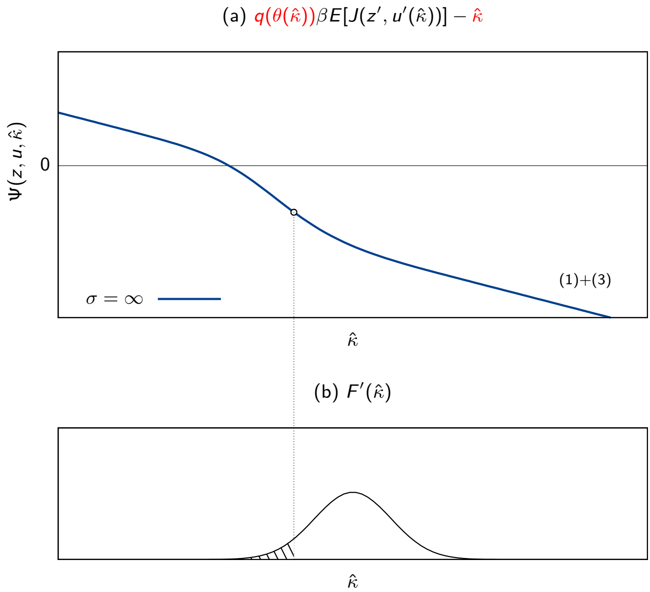
Sources of Multiplicity

There are two types of multiplicity:

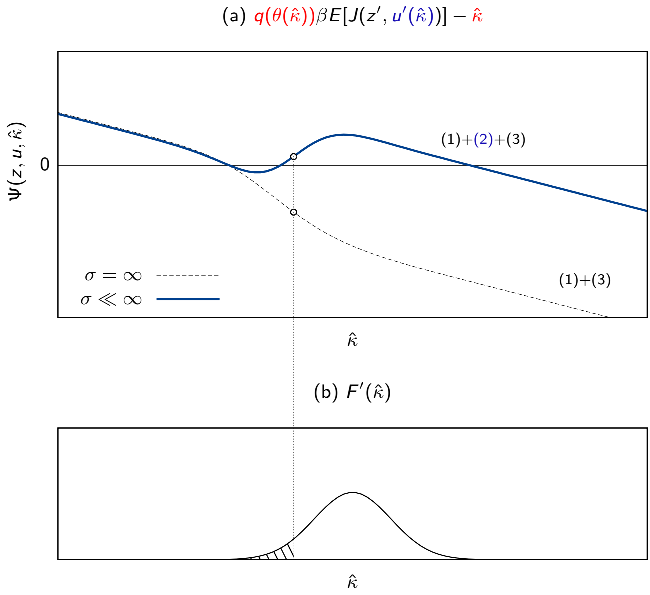
1. **Static**
   - Depending whether firms enter today or not
   - Possibly multiple solutions to the entry problem

(a) $q(\theta(\hat{\kappa}))\beta E[J(z', u'(\hat{\kappa}))] - \hat{\kappa}$





(a) $q(\theta(\hat{\kappa}))\beta E[J(z', u'(\hat{\kappa}))] - \hat{\kappa}$
$\Psi(z, u, \hat{\kappa})$
0
(1)+(3)
$\sigma = \infty$
$\hat{\kappa}$
(b) $F'(\hat{\kappa})$
$\hat{\kappa}$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$\hat{\kappa}$
(b) $F'(\hat{\kappa})$
$\hat{\kappa}$

Dynamic vs Static Multiplicity

There are two types of multiplicity:

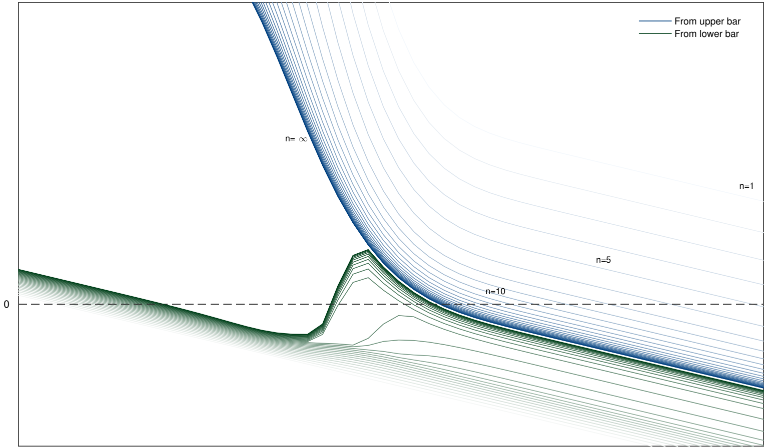
1. **Static**
   - Depending whether firms enter today or not
   - Possibly multiple solutions to the entry problem
2. **Dynamic**
   - Because jobs live several periods, expectations of future coordination matter
   - Multiple solutions to the Bellman equation
   - Usually strong: complementarities magnified by dynamics

Dynamic Multiplicity

- Usually difficult to say anything about dynamic multiplicity
- We can however say something about the set of equilibria
  - An equilibrium is summarized by value function J
  - The mapping for J is **monotone:**
    - *Tarski's fixed point theorem:* the set of fixed points is non-empty and admits a maximal and a minimal element.
    - They can be found numerically by iterating from upper and lower bounds of set
  - Provides an upper and lower bound on equilibrium value functions
    - If coincide $\Rightarrow$ uniqueness of equilibrium

Dynamic Multiplicity

$$\Psi\left(z, u, \hat{\kappa}\right) = q\left(\theta\left(\hat{\kappa}\right)\right)\beta E\left[J\left(z', u'\left(\hat{\kappa}\right)\right)\right] - \hat{\kappa}$$



Uniqueness

Proposition

If there exists $0 < \eta < 1 - (1-\delta)^2$ *such that for all* (u, θ),

$$\underbrace{\beta \overline{J}_u u p(\theta)\, \varepsilon_{p,\theta}}_{(2)} \leqslant \eta \frac{\kappa(\theta, u)}{q(\theta)} \left(\underbrace{\varepsilon_{q,\theta}}_{(1)} + \underbrace{\varepsilon_{\kappa,\theta}}_{(3)} \right),$$

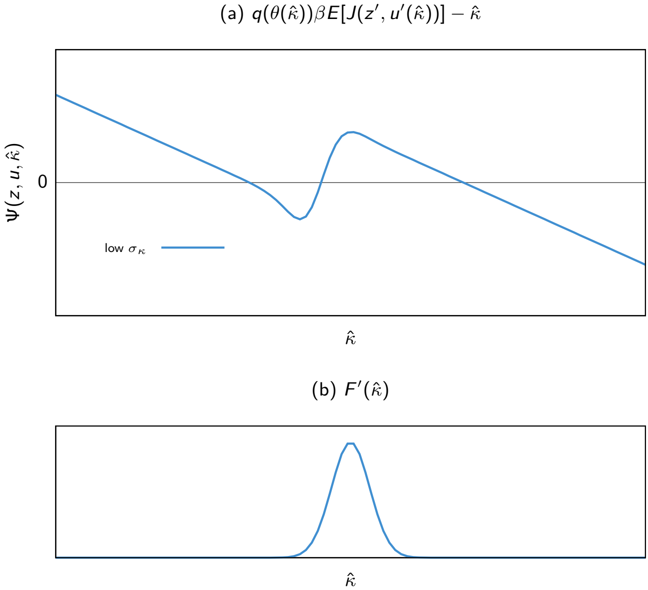
where $\varepsilon_{p,\theta} \equiv \frac{dp}{d\theta}\frac{\theta}{p(\theta)}$, $\varepsilon_{q,\theta} \equiv -\frac{dq}{d\theta}\frac{\theta}{q(\theta)}$, $\varepsilon_{\kappa,\theta} \equiv \frac{d\kappa}{d\theta}\frac{\theta}{\kappa}$, *then there exists a unique equilibrium if for all* (u, θ)

$$\frac{\beta}{1-\eta} \left| 1 - \delta - \gamma p(\theta) \left(1 + \frac{\varepsilon_{p,\theta}}{\varepsilon_{q,\theta} + \varepsilon_{\kappa,\theta}} \right) \right| < 1.$$

Corollary

1. *There is a unique equilibrium as* $\sigma \to \infty$ *(no complementarity).*
2. *For any* $\sigma > 1$, *there is a unique equilibrium as* $\sigma_\kappa \to \infty$.

Role of Heterogeneity



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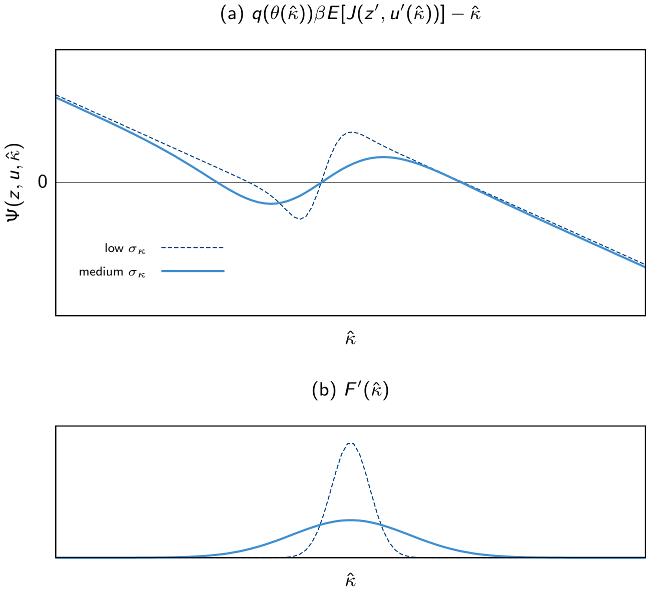

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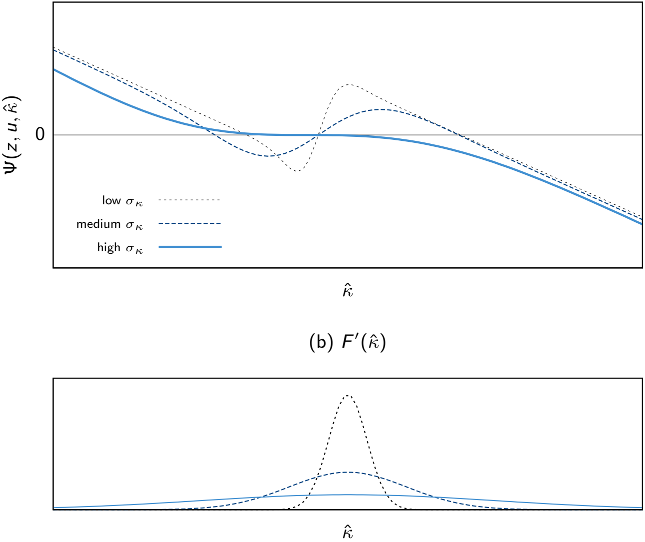
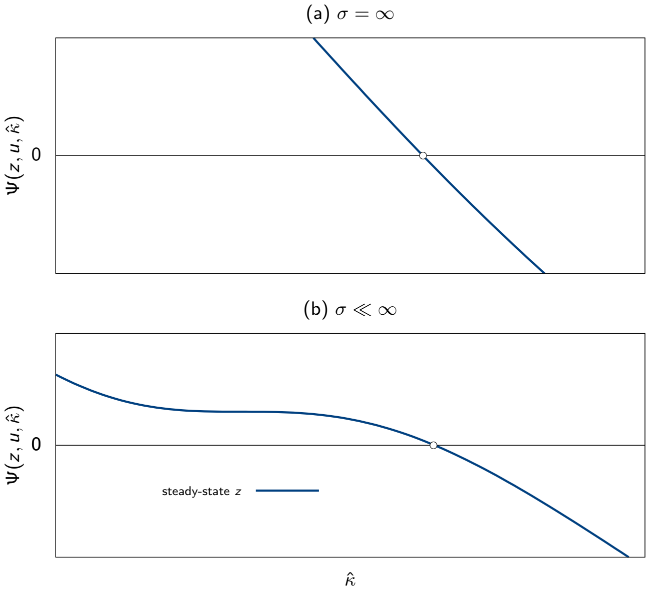


Non-linearities

- From now on, assume heterogeneity large enough to yield uniqueness
- Despite uniqueness, the model retains interesting features:
  - Highly non-linear response to shocks
  - Multiplicity of attractors/steady states

Non-linear Response to Shocks



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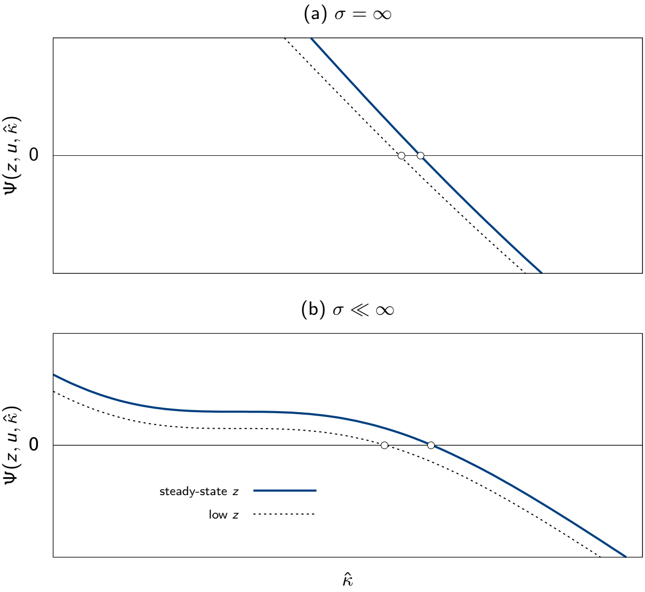

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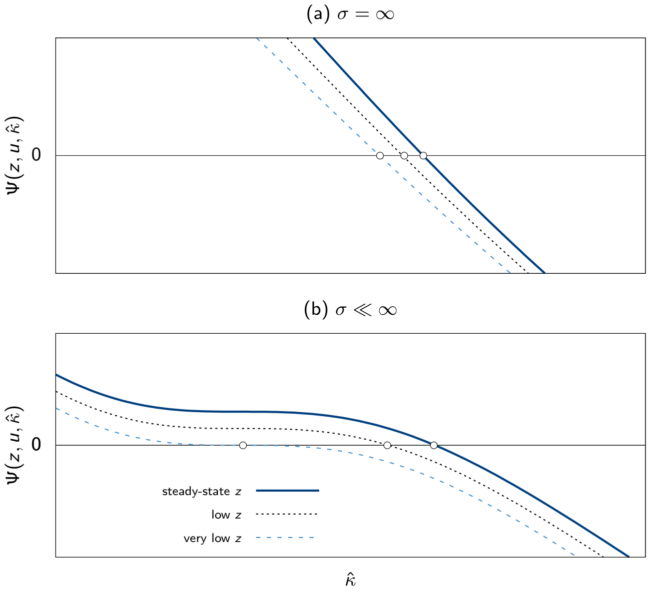

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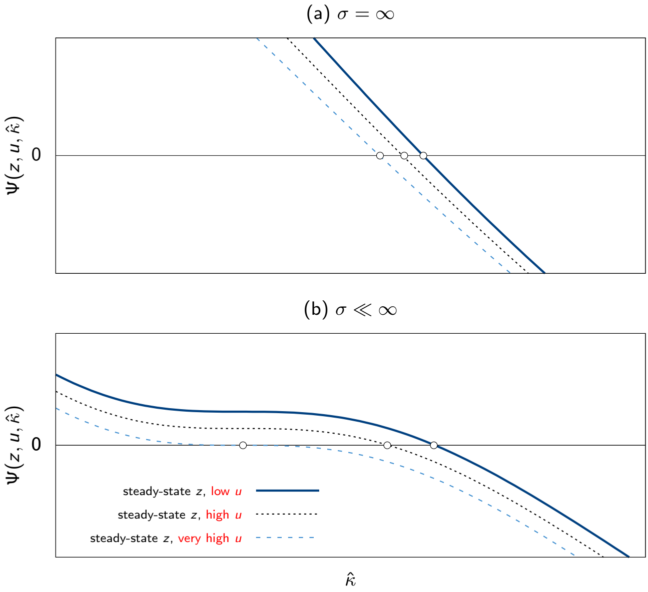

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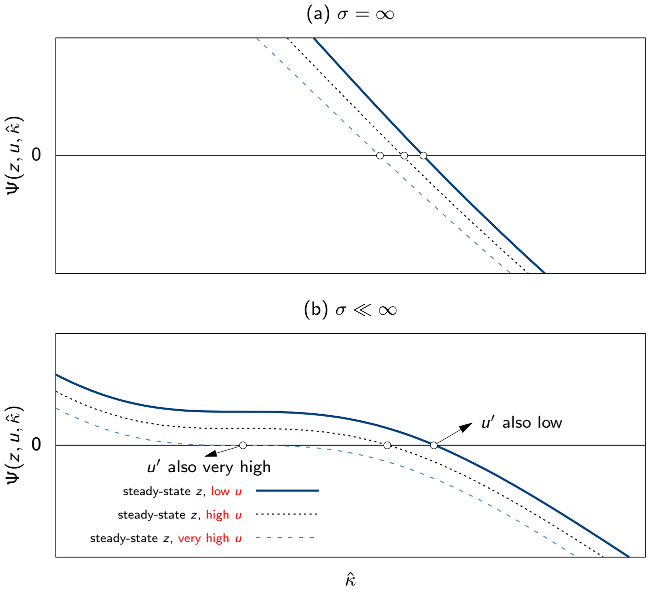
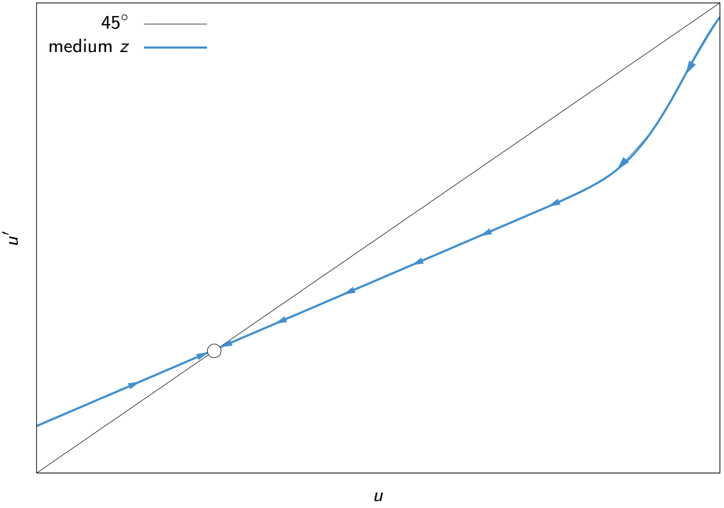


Non-linear Dynamics



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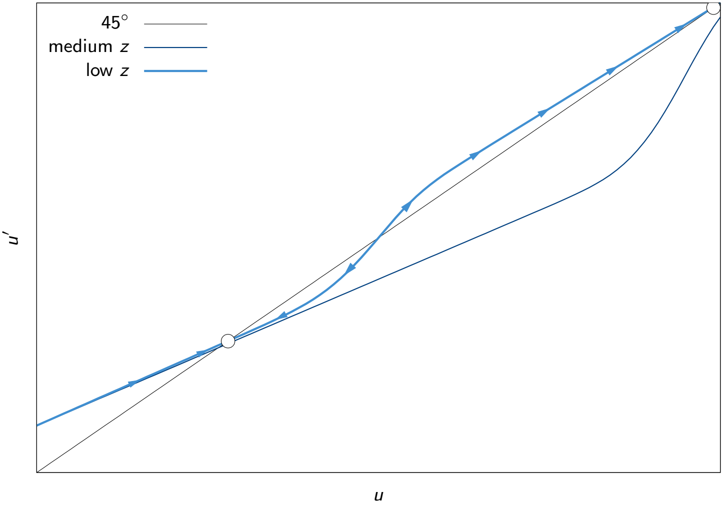

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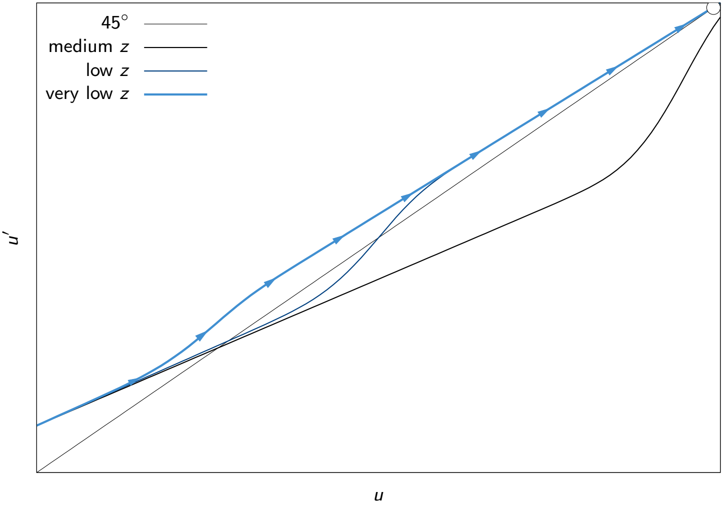

III. Quantitative Analysis

Calibration

- Period is ≈ 1 week (a twelfth of a quarter): $\beta = 0.988^{1/12}$
- Steady-state productivity $A = (1 - \bar{u})^{-1/(\sigma-1)}$
- Productivity process from data $\rho_z = 0.984^{1/12}$, $\sigma_z = \sqrt{1 - \rho_z^2} \times 0.05$
- Self-employed workers: average over last decades $s = 0.09$
- Matching function: $q(\theta) = (1 + \theta^{\mu})^{-1/\mu}$ and $p(\theta) = \theta q(\theta)$
- We get $\delta = 0.0081$ and $\mu = 0.4$ by matching
  - Monthly job finding rate of 0.45 (Shimer, 2005)
  - Monthly job filling rate of 0.71 (Den Haan et al., 2000)

Calibration

The elasticity of substitution σ is crucial for our mechanism

- Large range of empirical estimates
  - Establishment-level trade studies find $\sigma \approx 3$
    - Bernard et. al. AER 2003; Broda and Weinstein QJE 2006
  - Mark-up data says $\sigma \approx 7$
- We adopt $\sigma = 4$ as benchmark
  - Mark-ups are small ($\approx 2.4\%$) in our model because of bargaining and entry

Calibrating the distribution of costs $F(\kappa)$

- Hiring cost data from French firms (Abowd and Kramarz, 2003)

$$E(\kappa|\kappa < \hat{\kappa}) = 0.34 \text{ and } std(\kappa|\kappa < \hat{\kappa}) = 0.21$$

Markup Dispersion

Calibration

Two parameters left to calibrate

- Bargaining power γ
- Value of leisure for workers b

We target two moments

- Steady-state unemployment rate of 5.5%
- Elasticity of wages with respect to productivity of 0.8 (Haefke et al, 2013)

We find $\gamma = 0.2725$ and $b = 0.8325$

- Both numbers are well within the range used in the literature

We verify numerically that the equilibrium is unique.

- The mapping describing the equilibrium is monotone
- Starting iterations from the lower and upper bounds yield the same outcome

⇒ Uniqueness of the full dynamic equilibrium

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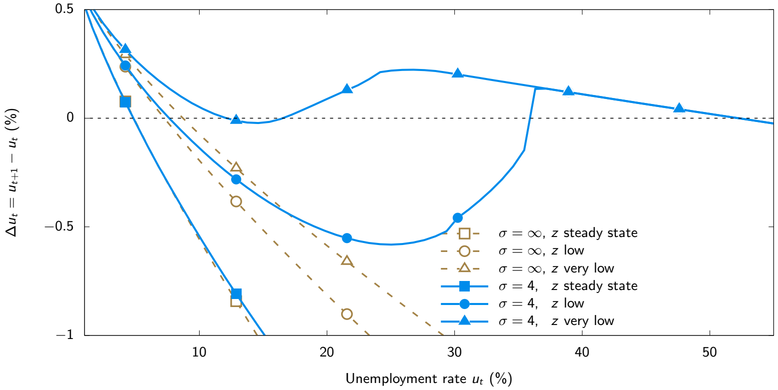
$\Rightarrow$ Uniqueness of the full dynamic equilibrium

Multiple steady states



Long-run moments - Volatility

Time-series properties after 1,000,000 periods

Standard Deviation	log u	log v	log θ
Data	0.26	0.29	0.44
Benchmark ($\sigma = 4$)	0.28	0.25	0.53
No complementarity ($\sigma = \infty$)	0.16	0.15	0.31

$\Rightarrow$ The mechanism generates additional volatility.

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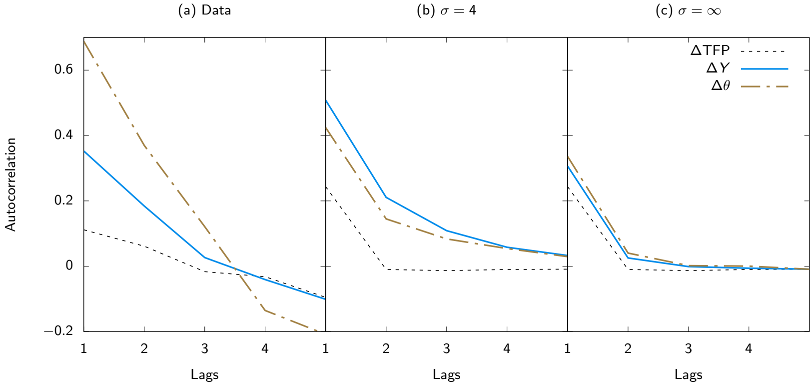
Long-run moments - Propagation

Autocorrelograms of growth in TFP, output and tightness



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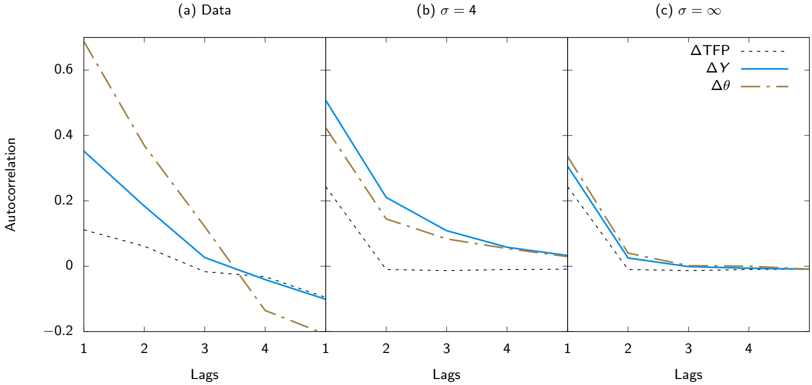

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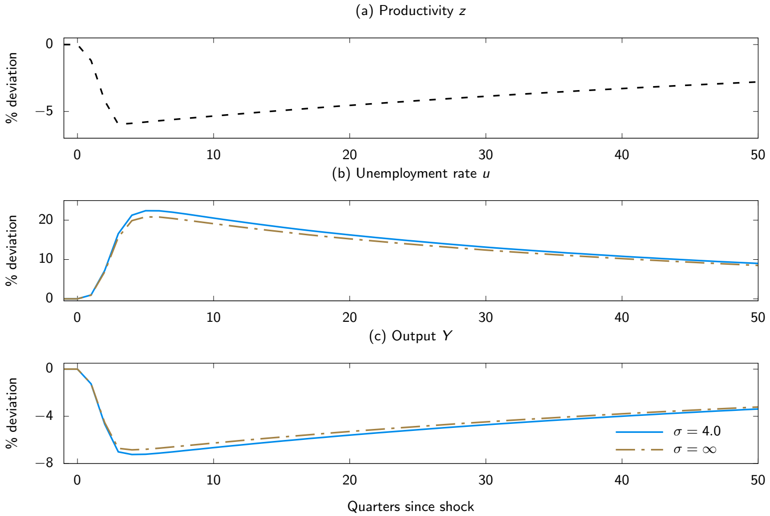
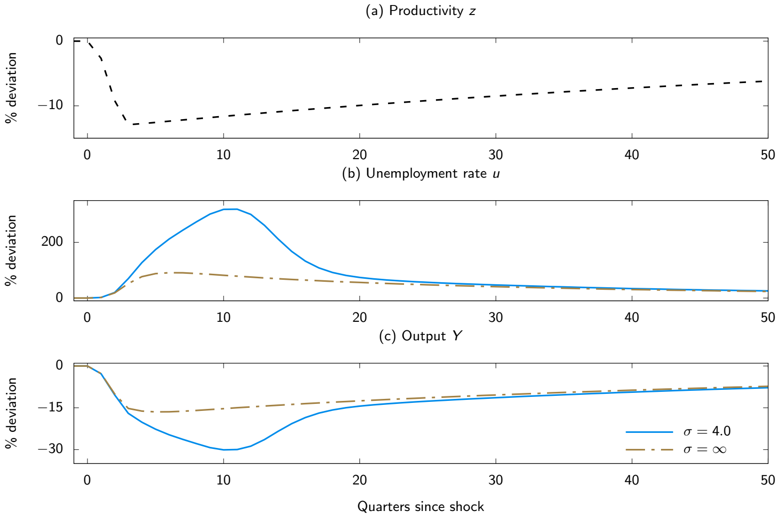
Impulse responses - **Small shock**



Notes: The innovation to z is set to -1 standard deviation for 2 quarters.

Impulse responses - **Large shock**



Notes: The innovation to z is set to -2.3 standard deviations for 2 quarters.

Conclusion

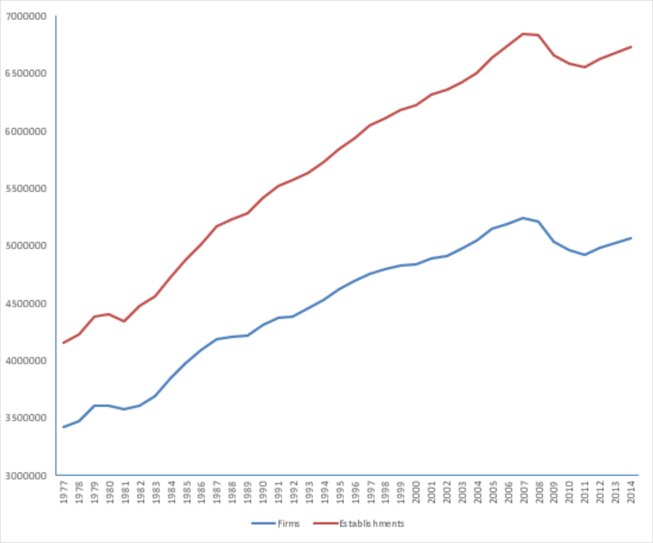
Summary

- We augment the DMP model with a demand channel
  - Demand channel amplifies and propagates shocks, in line with the data
  - Non-linear dynamics with possibility of multiple steady states
- We show uniqueness of the dynamic equilibrium when there is enough heterogeneity

Future research

- Optimal policy

Number of units of production



Return

Markup

In the model

$$\text{Markup} = \frac{\text{Unit price}}{\text{Unit cost}} = \frac{P_j}{w/Y_j} = \frac{P_j Y_j}{\gamma P_j Y_j + (1-\gamma)\, b + \gamma\beta\theta\hat{\kappa}}$$

- $P_j Y_j$ is normalized to one in the steady-state
- Calibration targets the steady-state values of $\hat{\kappa}$ and θ from the data

$\Rightarrow$ σ has no impact on steady-state markup

- Hagedorn-Manovskii (2008)
  - $\gamma = 0.052,\ b = 0.955,\ \bar{\kappa} = 0.584,\ \beta = 0.99^{1/12},\ \theta = 0.634$
  - Average markup $= 2.4\%$
- Shimer (2005)
  - $\gamma = 0.72,\ b = 0.4,\ \kappa = 0.213,\ \beta = 0.988,\ \theta = 0.987$
  - Average markup $= 1.9\%$

Return

Markup

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  - Average markup = 1.9%

Return

Calibration dispersion κ

Calibrating the distribution of costs $F(\kappa)$

- Hiring cost data from French firms (Abowd and Kramarz, 2003)
  - Assume:
  $$\text{Hiring cost} = D \times w$$
  where D, the cost of hiring per unit of wage, is iid.
  - Then:
  $$E(\kappa|\kappa < \hat{\kappa}) = 0.34 \text{ and } std(\kappa|\kappa < \hat{\kappa}) = 0.21$$
- Find the steady-state value of $\hat{\kappa}$ from steady-state free-entry condition
  - Assume $F(\kappa)$ is normal $\rightarrow$ $F(\kappa)$ is fully characterized
- We find $M = \bar{v}/F(\hat{\kappa}) = 3.29$ using steady-state $\bar{v}$ from data and with

$$\hat{\kappa} = q(\bar{\theta})\,\beta \frac{(1-\gamma)(1-b)}{1-\beta\left(1-\delta-\gamma p(\bar{\theta})\right)}$$

Return