

Herding through Booms and Busts

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 - ▶ New technology accompanied by massive investment
 - ▶ Followed by a sharp contraction in macro aggregates
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- A prominent view is that these cycles are expectations driven (Pigou, 1927)
 - ▶ Most of the literature: *exogenous* movements in expectations
 - ▶ **But expectations have a life of their own**
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 - How can we explain the evolution of beliefs from optimistic to pessimistic?

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 - ▶ **But expectations have a life of their own**
 - Why do people become optimistic in the first place?
 - How can we explain the evolution of beliefs from optimistic to pessimistic?
- An important driver of expectations is the *observation of others*
 - ▶ Investment begets investment ⇒ **herding**

- We embed a model of **rational herding** into a business cycle framework
 - ▶ Agents learn from observing the investment behavior of others (**social learning**)
 - ▶ People can collectively fool themselves into thinking they're in a boom until they realize their mistake (bust)

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- Boom-bust cycles as **false-positives**
 - ▶ New technology arrives with uncertain quality
 - ▶ Agents have private information and observe the investment decisions of others
 - ▶ Importantly, we assume that there is **common noise** in private signals
 - correlation of beliefs due to agents having similar sources of information
 - ▶ High investment indicates either
 - **good technology**, or
 - **bad technology** but agents received **optimistic private signals**.

- Development of a boom-bust cycle:
 - ▶ Unusually large realization of common noise may send the economy on self-confirming boom
 - agents have optimistic private signals and invest a lot
 - high investment is mistakenly attributed to technology being good
 - agents invest more which seems to confirm initial assessment
 - more investment \Rightarrow more optimism \Rightarrow more investment...
 - ▶ But agents are rational and information keeps arriving, so probability of false-positive state rises
 - at some point, most pessimistic agents stop investing
 - suddenly, high beliefs are no longer confirmed by experience
 - sharp reversal in beliefs and collapse of investment \Rightarrow bust
 - truth is learned in the long run

- **Results**

- ▶ Model can produce endogenous boom-bust cycles
- ▶ Theory has predictions on **bubble-like** phenomena over the business cycle
 - When/why they arise, under what conditions, at what frequency
 - When/why they burst without exogenous shock
- ▶ Since cycle is endogenous, policies are particularly powerful
 - Policies can affect the **boom duration/amplitude** and **timing of the burst**
 - Optimal policies (tax) leans against the wind, monetary policy ill-suited
- ▶ Quantification
 - Theory can generate **realistic, sizable** boom-bust cycles
 - Endogenous boom-bust cycles **above** and **below** trend

- **Bubbles**

- ▶ Macro: rational bubbles (Tirole, 1985; Martin and Ventura, 2012; Galí, 2014...), financial frictions (Kocherlakota, 1992; Miao and Wang, 2013, 2015...)
 - ⇒ specific sequence of exogenous sunspots
- ▶ Finance: agency problem (Allen and Gale, 2000;...), heterogeneous beliefs (Harrison and Kreps, 1978; Allen et al., 1993), asymmetric information (Abreu and Brunnermeier, 2003;...)
 - ⇒ price \neq fundamental, dynamics not the focus

- **News/noise-driven cycle**

- ▶ Beaudry and Portier (2004, 2006, 2014), Jaimovich and Rebelo (2009), Lorenzoni (2009), Schmitt-Grohé and Uribe (2012), Blanchard, Lorenzoni and L'Huillier (2013), etc.
 - ⇒ Our theory can endogenize the information process that leads to news-driven cycles

- **Herding**

- ▶ Banerjee (1992), Bikhchandani et al. (1992), Avery and Zemsky (1998), Chamley (2004)
- ▶ Drawbacks of early herding models:
 - Rely crucially on agents moving sequentially and making binary decisions
 - Boom-busts only arise for specific sequence of events and **particular ordering of people**
- ▶ **This paper**
 - Relax sequentiality of moves and binarity of decisions (\Rightarrow easier intro to standard models)
 - Boom-bust cycles arise endogenously **after a single impulse shock** (\Rightarrow natural evolution of beliefs in the presence of common noise)

1. Simplified learning model
2. Business-cycle model with herding

- Simple, abstract model
- Time is discrete $t = 0, 1, \dots, \infty$
- Unit continuum of risk neutral agents indexed by $j \in [0, 1]$

- Agents choose whether to invest or not, $i_{jt} = 1$ or 0
 - ▶ Investing requires paying the cost c
- Investment technology has common return

$$R_t = \theta + u_t$$

with:

- ▶ Permanent component $\theta \in \{\theta_H, \theta_L\}$ with $\theta_H > \theta_L$, drawn once-and-for-all
- ▶ Transitory component $u_t \sim \text{iid } F^u$

- Agents receive a private signal s_j drawn from distributions with pdf $f_{\theta+\xi}^s(s_j)$
 - ▶ ξ is some common noise drawn from CDF F^ξ
 - captures the fact that agents learn from common sources (media, govt)
- **Example:** $f_{\theta+\xi}^s \sim \mathcal{N}(\theta + \xi, \sigma_s^2)$

$$s_j = \theta + \xi + v_j \text{ where } v_j \sim \text{iid } \mathcal{N}(0, \sigma_s^2)$$

- In addition, all agents observe public signals
 - ▶ return on investment R_t
 - ▶ measure of investors m_t (social learning)

- Measure of investors is

$$m_t = \int_0^1 i_{jt} dj + \varepsilon_t$$

where $\varepsilon_t \sim \text{iid } F^m$ captures informational noise or noise traders

- Measure m_t is an endogenous nonlinear aggregator of private information
 - ▶ how much information is released varies over time

Simple timing:

- At date $t = 0$: θ , ξ and the s_j 's are drawn once and for all
- At date $t \geq 0$,
 1. Agent j chooses whether to invest or not
 2. Production takes place
 3. Agents observe $\{R_t, m_t\}$ and update their beliefs

- Beliefs are **heterogeneous**
- Denote **public information to an outside observer** at beginning of period t

$$\begin{aligned}\mathcal{I}_t &= \{R_{t-1}, m_{t-1}, \dots, R_0, m_0\} \\ &= \{R_{t-1}, m_{t-1}\} \cup \mathcal{I}_{t-1}\end{aligned}$$

- Multiple sources of uncertainty so must keep track of **joint distribution** of public beliefs:

$$\Lambda_t(\tilde{\theta}, \tilde{\xi}) = Pr(\theta = \tilde{\theta}, \xi = \tilde{\xi} | \mathcal{I}_t)$$

- The information set of agent j is

$$\mathcal{I}_{jt} = \mathcal{I}_t \cup \{s_j\}$$

- Recover **individual beliefs** Λ_{jt} using Bayes' law over Λ_t and s_j

Learning Model: Characterizing Beliefs

- For ease of exposition, simplify aggregate uncertainty to three states

$$\omega = (\theta, \xi) \in \left\{ \underbrace{(\theta_L, 0)}_{\text{bad}}, \underbrace{(\theta_H, 0)}_{\text{good}}, \underbrace{(\theta_L, \bar{\xi})}_{\text{false-positive}} \right\} \text{ with } \theta_L < \theta_L + \bar{\xi} < \theta_H$$

- ▶ $\omega = (\theta_L, \bar{\xi})$ is the **false-positive** state: technology is low, but agents receive unusually positive news
- Just need to keep track of two state variables (p_t, q_t)

$$p_t \equiv \Lambda_t(\theta_H, 0) \text{ and } q_t \equiv \Lambda_t(\theta_L, \bar{\xi})$$

- Can recover private beliefs $p_{jt} \equiv p_j(p_t, q_t, s_j)$ and $q_{jt} \equiv q_j(p_t, q_t, s_j)$ from Bayes' law

▶ Details

- Agents invests iff

$$E_{jt} [R_t | \mathcal{I}_{jt}] \geq c$$

- Under **MLRP** for f^s , optimal investment decision is a cutoff rule $s^*(p_t, q_t)$:

$$i_{jt} = 1 \Leftrightarrow s_j \geq s^*(p_t, q_t)$$

- The measure of investing agents is

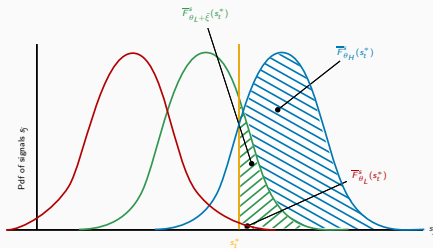
$$m_t = \overline{F}_{\theta+\xi}^s (s^* (p_t, q_t)) + \varepsilon_t$$

- ▶ $\overline{F}_{\theta+\xi}^s (s_j)$ is complementary CDF of private signal s_j
- ▶ Since $s^* (p_t, q_t)$ and $\{\overline{F}_\omega^s\}_{\omega \in \Omega}$ known to all agents, m_t is a noisy signal about $\theta + \xi$

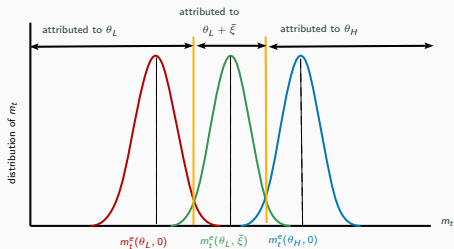
▶ Bayesian updating

Endogenous Learning: 3-state example

- In the 3-state example, only three measures m_t are possible (up to ε_t)



- Distributions of $m_t = \bar{F}^S(\hat{s}_t) + \varepsilon_t$ in the 3 states of the world



State-dependent Informativeness

- Informativeness of m_t varies over time



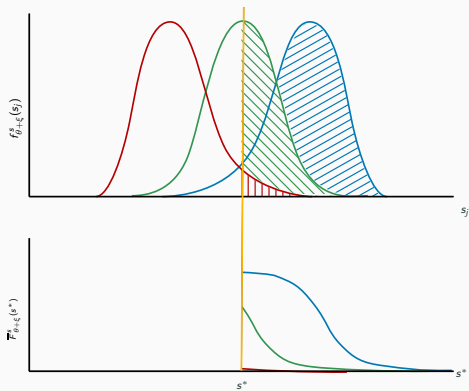
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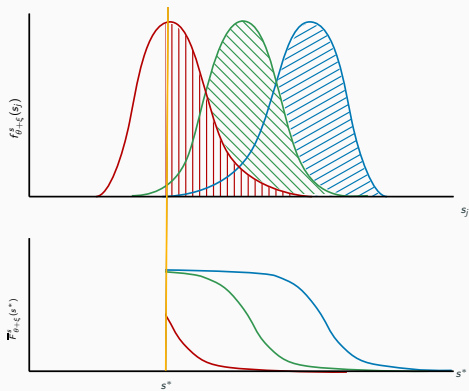
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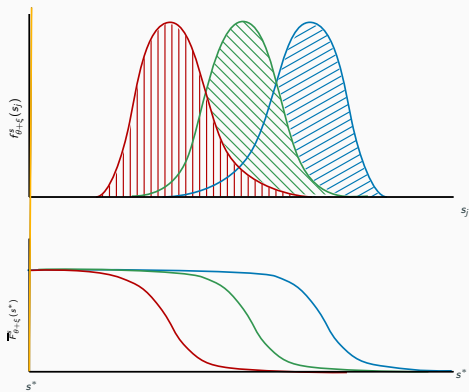
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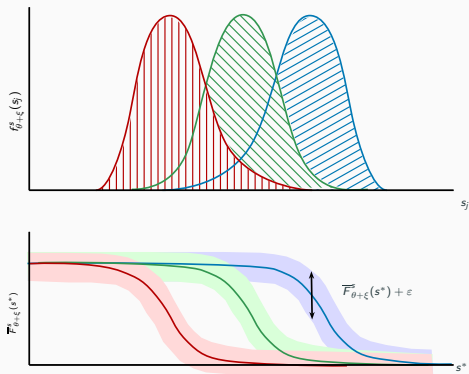
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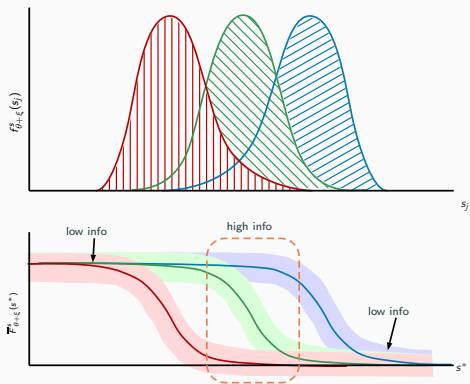
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Why is this interesting?

- **Asymmetry**
 - ▶ helps explain why booms are slow to take off when few people invest and crashes are sudden
- **Persistence**
 - ▶ “bubbly situations” can persist for a long time when agents herd on same action (information cascade)
- **Policy**
 - ▶ some policy intervention may suddenly release information and trigger bust
 - ▶ motivates leaning-against-the-wind policies

- **Parametrization**

- ▶ Fundamentals: $\theta_h = 1.0$, $\theta_l = 0.5$, $\bar{\xi} = 0.4$
- ▶ Gaussian signals:

$$s_j = \theta + \xi + v_j \text{ with } v_j \sim \mathcal{N}(0, \sigma_v^2)$$

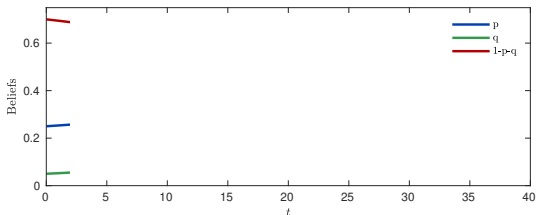
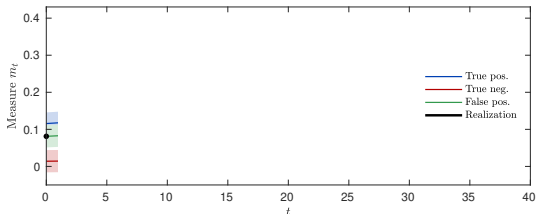
- ▶ Priors: $P(\theta_l, \bar{\xi}) \ll P(\theta_h, 0)$

▶ True negative

▶ True positive

Simulations: False Positive ($\theta_1, \bar{\xi}$)

- Boom phase

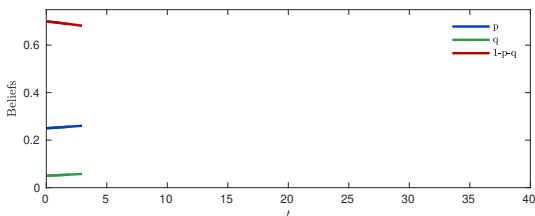
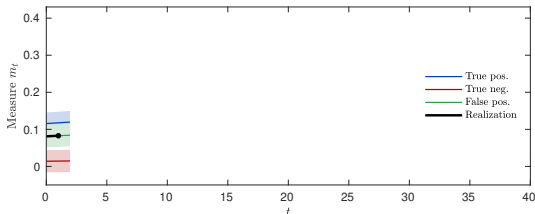


- Mechanism

- ▶ high investment rates consistent with true and false positive $\Rightarrow p$ and q rise progressively
- ▶ for initial q_0 sufficiently low, most of it is attributed to high state (p dominates)

Simulations: False Positive ($\theta_1, \bar{\xi}$)

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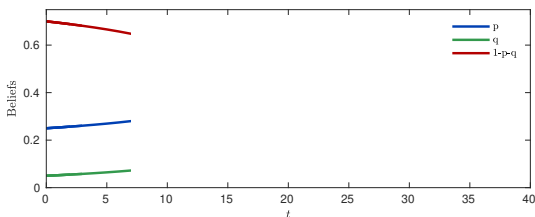
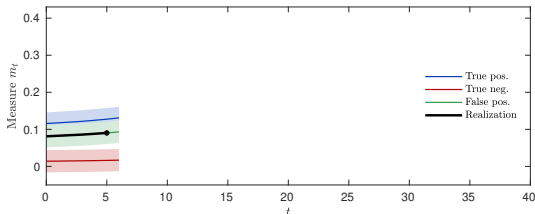


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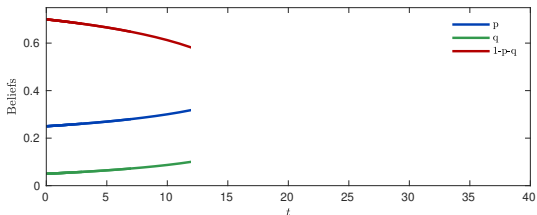
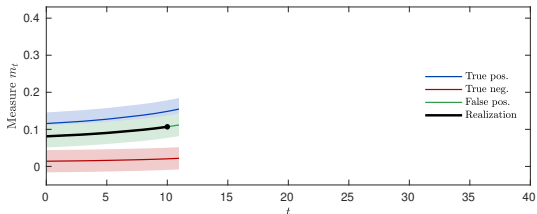


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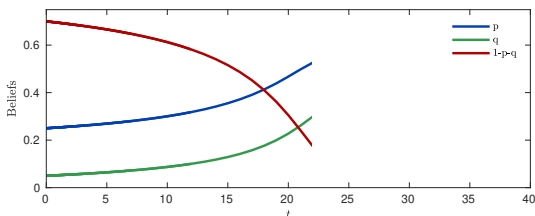
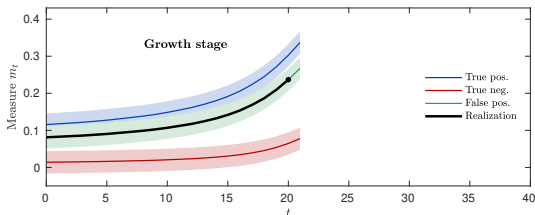


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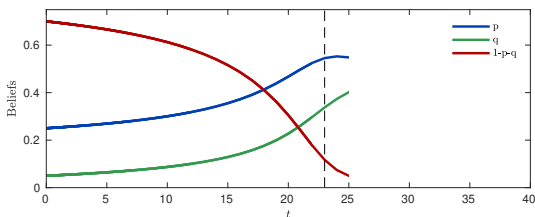
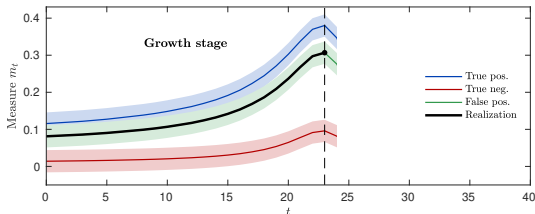


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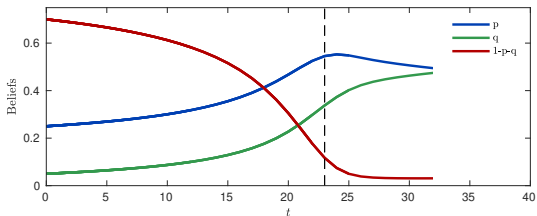
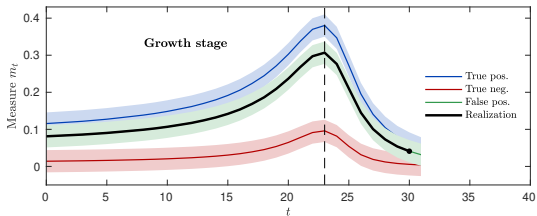


• Mechanism

- ▶ when q high enough, some investors leave the market, releasing more information
- ▶ early exit of investors incompatible with high state \Rightarrow quick collapse of investment

Simulations: False Positive ($\theta_1, \bar{\xi}$)

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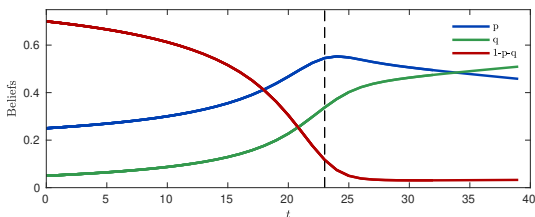
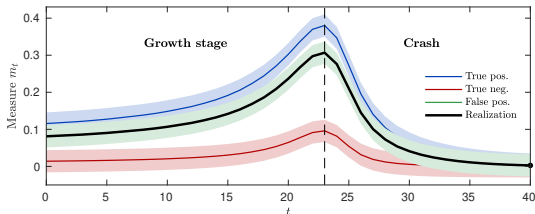


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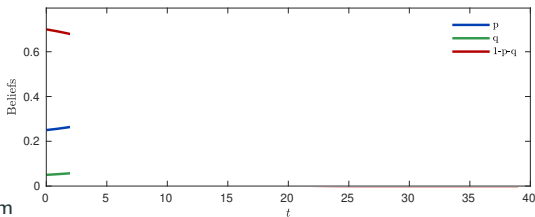
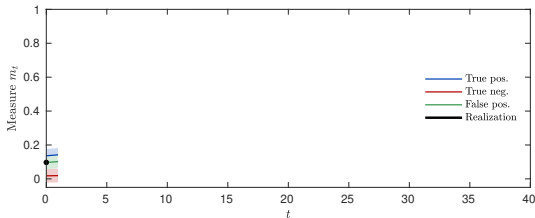


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Simulations: False Positive ($\theta_I, \bar{\xi}$)

- With larger shock, an **information cascade** may arise

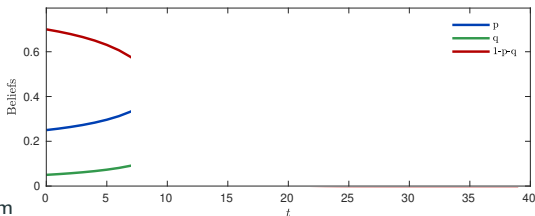
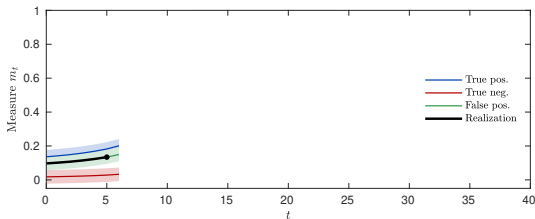


- Mechanism

- ▶ p is so high that almost everyone invests, releasing close to no information
- ▶ because information not exactly 0, q slowly rises in the background

Simulations: False Positive ($\theta_1, \bar{\xi}$)

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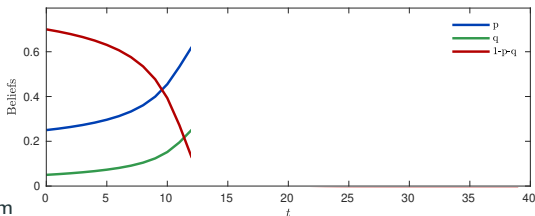
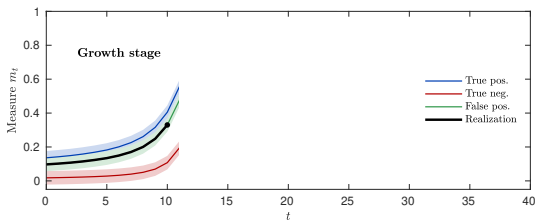


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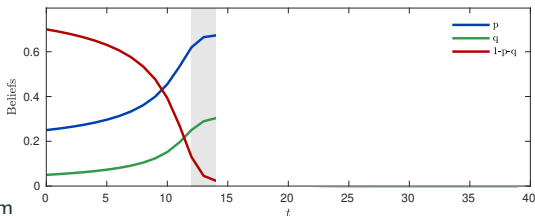
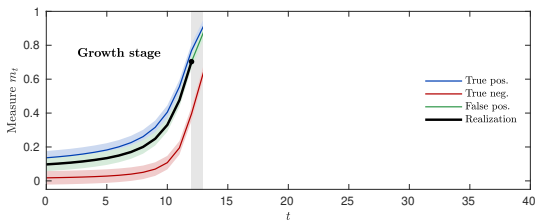


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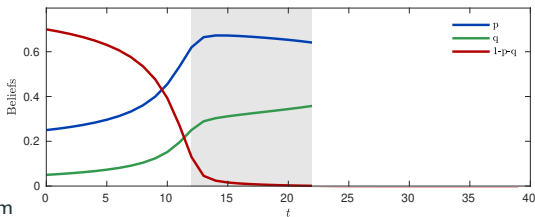
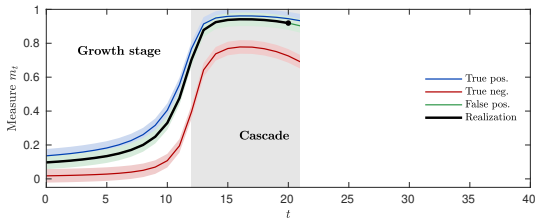


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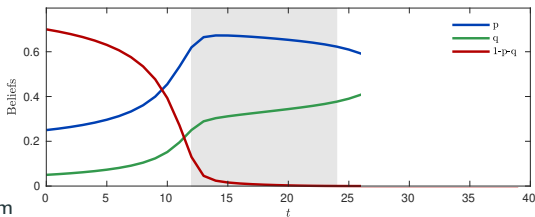
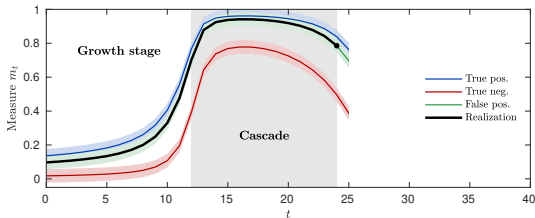


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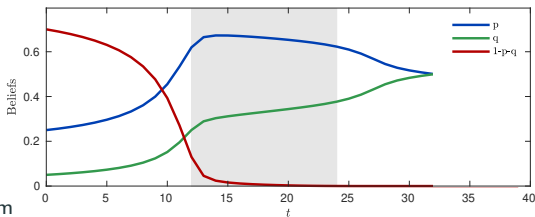
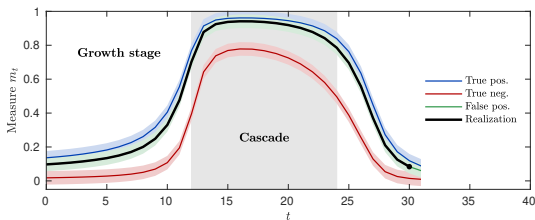


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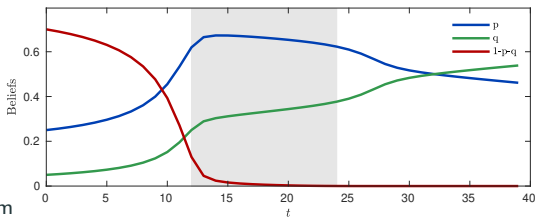
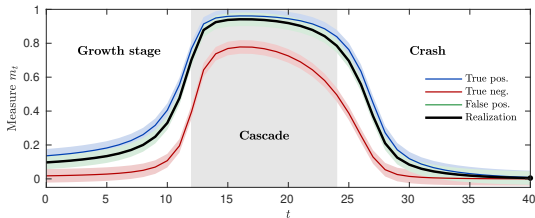


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- Allow ξ to take a continuum of values [▶ Go](#)
 - ▶ Results survive
 - ▶ **Proposition:** there always exists a threshold $\underline{\xi}$ such that $\xi \geq \underline{\xi}$ triggers a boom and bust episode.
- Planner's problem [▶ Go](#)
 - ▶ The equilibrium is inefficient
 - ▶ Planner adopts lean-against-the-wind policies

1. Learning model
2. Business-cycle model with herding

Herd-driven Business Cycle Model

- Objective

- ▶ How do boom-and-bust in beliefs lead to general macroeconomic expansion, followed by a below-trend contraction?
- ▶ Full-fledge macro model amenable for quantification and policy analysis

- Parsimonious NK DSGE model with [▶ Details](#)

1. Dynamic arrival of new technologies and **technology choice**
2. Entrepreneurs choose **new vs. old** technology and learn from measure of tech adopters
3. **Two types of capital**: Traditional (T) and Information Technology (IT)
 - IT investment is required to enjoy the new technology
4. **Nominal rigidities**
 - Study impact of monetary policy

- Mechanism

- ▶ Entrepreneurs choose **new vs. old** technology and agents learn from measure of tech adopters
 - ▶ Boom fueled by **build-up of IT capital** and **positive wealth effect** on consumption
 - ▶ Belief reversal causes sudden realization of misallocation in investments
- ⇒ negative wealth effect and collapse of IT investment causing recession

- **Calibration**

[▶ Details](#)

- ▶ Based on the dot-com boom-bust episode
- ▶ Uses data from the Survey of Professional Forecaster to discipline beliefs

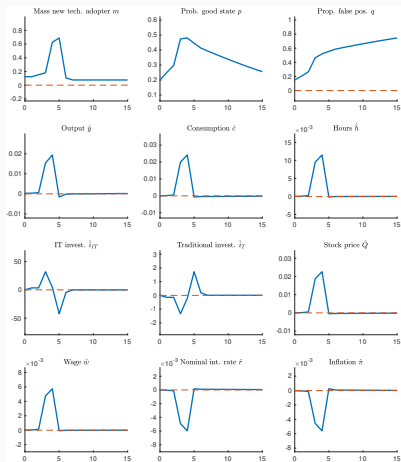
IRF to False-Positive

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- Impulse response: false positive $(\theta, \xi) = (\theta_I, 0.75(\theta_h - \theta_I))$



- Mechanism

- ▶ Positive wealth effect $c \nearrow$,
- ▶ Build-up of future IT capital $i^{IT} \nearrow$
- ▶ Anticipation of future productivity growth $\Rightarrow \pi \searrow, r \searrow$
- ▶ Aggregate demand $\nearrow \Rightarrow y \nearrow, h \nearrow$

- Quantitative

- ▶ Endogenous boom-bust with positive comovement between c, i, h and y
- ▶ But boom-bust may arise at high probability (benchmark 15% $\gg 10^{-6}$ (Avery and Zemsky, 1998))

- Govt policies are powerful in this setup
 - ▶ **Learning externality:** agents do not internalize that investment affects release of info
 - ▶ Since cycle is endogenous, policies can **substantially dampen** boom-busts
- Monetary policy that **leans-against-the-wind:** [▶ Details](#)
 - ▶ May succeed in dampening fluctuations
 - ▶ But barely affects the **new vs. old technology** trade-off to take care of learning externality
 - ▶ Stabilizes boom-bust in the new technology at the expense of other sector

- Introduce herding phenomena as a potential **source of business cycles**
- We have proposed a business cycle model with herding
 - ▶ people can collectively fool themselves for extended period of time
 - ▶ endogenous boom-bust cycles patterns after unusually large noise shocks
 - ▶ the model has predictions on the **timing and frequency** of such phenomena
- Quantitatively, such crises can arise with relatively **high probability** despite fully rational agents
- Provides rationale for **leaning-against-the-wind** policies which can substantially dampen fluctuations

- Private beliefs (p_{jt}, q_{jt}) are given by Bayes' law:

$$p_{jt} \equiv p_j(p_t, q_t, s_j) = \frac{p_t f_{\theta_H}^s(s_j)}{p_t f_{\theta_H}^s(s_j) + q_t f_{\theta_L + \bar{\xi}}^s(s_j) + (1 - p_t - q_t) f_{\theta_L}^s(s_j)}$$

$$q_{jt} \equiv q_j(p_t, q_t, s_j) = \frac{q_t f_{\theta_L + \bar{\xi}}^s(s_j)}{p_t f_{\theta_H}^s(s_j) + q_t f_{\theta_L + \bar{\xi}}^s(s_j) + (1 - p_t - q_t) f_{\theta_L}^s(s_j)}$$

- Under MLRP, individual beliefs p_j are monotonic in s_j

$$\frac{\partial p_j}{\partial s_j}(p_t, q_t, s_j) \geq 0$$

Monotone Likelihood Ratio Property

- **Assumption:** F_x^s satisfies *monotone likelihood ratio property* (MLRP)
 - ▶ *i.e.:* a higher s signals a higher $\theta + \xi$

$$x_2 > x_1 \text{ and } s_2 > s_1 \Rightarrow \frac{f_{x_2}^s(s_2)}{f_{x_1}^s(s_2)} \geq \frac{f_{x_2}^s(s_1)}{f_{x_1}^s(s_1)} \quad (\text{MLRP})$$

- Satisfied by many standard distributions like $f_\theta^s \sim N(\theta, \sigma^2)$, etc.

◀ Return

- After observing m_t , public beliefs are updated

$$p_{t+1} = \frac{p_t f^m \left(m_t - \bar{F}_{\theta_H}^s (s_t^*) \right)}{\Omega}$$

and

$$q_{t+1} = \frac{q_t f^m \left(m_t - \bar{F}_{\theta_L + \bar{\xi}}^s (s_t^*) \right)}{\Omega}$$

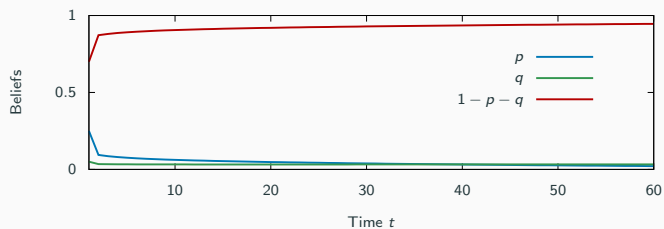
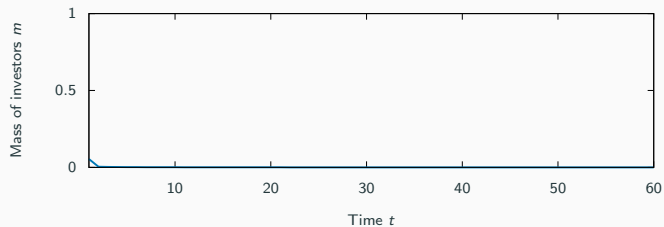
where

$$\Omega = p_t f^m \left(m_t - \bar{F}_{\theta_H}^s (s_t^*) \right) + q_t f^m \left(m_t - \bar{F}_{\theta_L + \bar{\xi}}^s (s_t^*) \right) + (1 - p_t - q_t) f^m \left(m_t - \bar{F}_{\theta_L}^s (s_t^*) \right)$$

- Similar updating rule with exogenous signal $R_t = \theta + u_t$

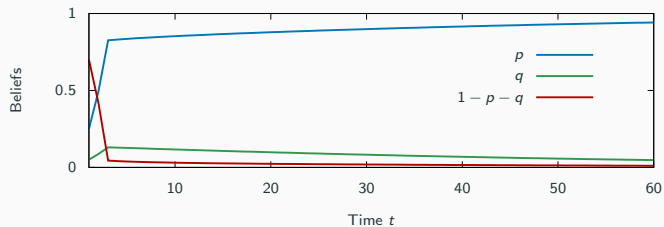
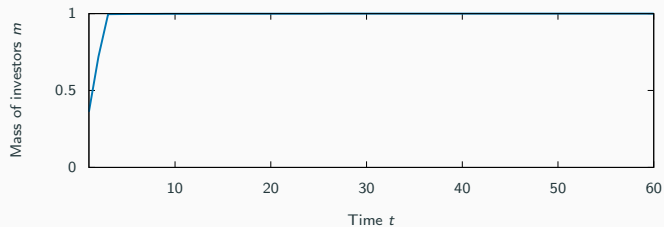
◀ Return

Simulations: True Negative ($\theta_I, 0$)



Return

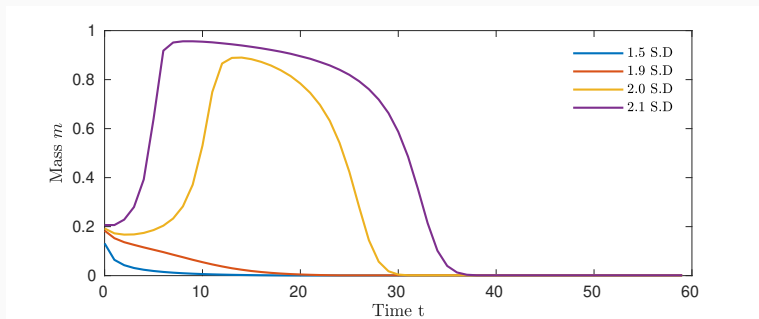
Simulations: True Positive ($\theta_h, 0$)



Return

- Previous simulations may look knife-edge
 - ▶ require state $(\theta_I, \bar{\xi})$ to be infrequent and resemble $(\theta_H, 0)$
- We now allow ξ to take a continuum of values
- **Take-away:**
 - ▶ small shocks (<1 SD) are quickly learned,
 - ▶ but unusually large shocks lead to boom-bust pattern

- True fundamental ($\theta_I = 0, \xi = \text{multiple of } \sigma_\xi$)

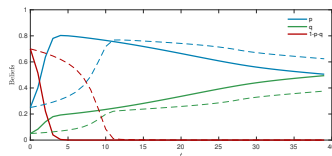
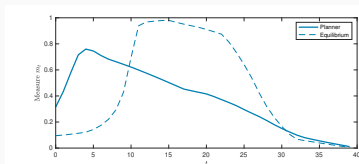


Proposition

In the Gaussian case, for θ and ξ independent and R_t sufficiently uninformative, there always exists a threshold $\underline{\xi}$ such that $\xi \geq \underline{\xi}$ triggers a boom and bust episode.

← Return

- **Information externality:** agents do not internalize how investment affects the release of information
 - ▶ They invest too much in a boom (too little in a negative boom)
- We study the constrained-efficient planning problem ▶ Go
 - ▶ Optimal policy **leans against the wind** to maximize collect of information
 - ▶ Implementation with investment tax/subsidy
 - ▶ Stabilizing “bubbles” comes at the cost of slowing good booms



◀ Return

- We adopt the welfare criterion from Angeletos and Pavan (2007)

$$V(p, q) = \max_{\hat{s}} E_{\theta, \xi} \left[\int_{\hat{s}} E[\theta - c | \mathcal{I}_j] dj + \gamma V(p', q') | \mathcal{I} \right]$$

where \mathcal{I} is public info and \mathcal{I}_j is individual info

- Crucially, the planner **understands how \hat{s} affects evolution of beliefs**

Business Cycle Model: Summary

- Four types of agents:
 - ▶ Households, **Entrepreneurs**, Retailers and Monetary Authority
- Three sectors: entrepreneur sector, retail sector and final good
- Two types of capital: **IT** vs. **traditional**
- Entrepreneurs choose between two technologies: **new** vs. **old**
 - ▶ new technology more intensive in IT capital

$$Y_{it} = A_{it} \left(\omega_i \left(K_i^{IT} \right)^{\frac{\zeta-1}{\zeta}} + (1 - \omega_i) \left(K_i^T \right)^{\frac{\zeta-1}{\zeta}} \right)^\alpha \frac{\zeta}{\zeta-1} (L_{it})^{1-\alpha}, i \in \{n, o\}$$

- Herding in **technology adoption**:
 - ▶ $\theta \in \{\theta_H, \theta_L\}$ is drawn and entrepreneurs receive private signals (+ common noise ξ)
 - Initially $A_{nt} = A_{ot}$ until technology matures (prob. λ) then $A_{nt} = \theta$.
 - ▶ Measure of entrepreneurs using new technology

$$m_t = (1 - \mu) \bar{F}_{\theta+\xi}^s (s_t^*) + \mu \varepsilon_t$$

where μ = measure of noise entrepreneurs

- ▶ Entrepreneurs learn from observing m_t

Business Cycle Model: Population

- Agents:
 - ▶ Households [▶ Details](#)
 - ▶ Retailers and monetary authority [▶ Details](#)
 - ▶ Entrepreneurs
- Three sectors: entrepreneur sector, retail sector and final good
 - ▶ **Entrepreneur sector:** technology choice, no nominal rigidities
 - ▶ **Retail sector:** buys the bundle of goods from entrepreneurs, subject to nominal rigidities
 - ▶ **Final good:** bundle of retail goods used for consumption and investment

- Unit measure of entrepreneurs indexed by $j \in [0, 1]$
 - ▶ monopolistic producers of a single variety
- At any date, there is a traditional technology (“old”) to produce varieties

$$Y_{jt}^o = A^o \left(\omega_o \left(K_o^{IT} \right)^{\frac{\zeta-1}{\zeta}} + (1 - \omega_o) \left(K_o^T \right)^{\frac{\zeta-1}{\zeta}} \right)^{\alpha \frac{\zeta}{\zeta-1}} \left(L_{jt}^o \right)^{1-\alpha}$$

- With probability η , an innovative technology arrives (“new”)

$$Y_{jt}^n = A_t^n \left(\omega_n \left(K_n^{IT} \right)^{\frac{\zeta-1}{\zeta}} + (1 - \omega_n) \left(K_n^T \right)^{\frac{\zeta-1}{\zeta}} \right)^{\alpha \frac{\zeta}{\zeta-1}} \left(L_{jt}^n \right)^{1-\alpha}$$

where

$$\omega_n > \omega_o$$

- The new technology needs to mature to become fully productive

$$A_t^n = \begin{cases} A^o & \text{before maturation} \\ \theta & \text{after} \end{cases}$$

- The new technology matures with probability λ per period
- The true productivity θ is high or low $\theta \in \{\theta_H, \theta_L\}$ with $\theta_H > \theta_L$

- Each period, entrepreneurs choose which technology to use
 - ▶ for simplicity, assume no cost of switching so problem is static
 - ▶ denote m_t the measure of entrepreneurs that adopt the new technology
- A fraction μ of entrepreneurs is clueless when it comes to technology adoption
 - ▶ “noise entrepreneurs”
 - ▶ random fraction ε_t adopts the new technology

- At $t = 0$, all entrepreneurs receive a private signal about θ from pdf $f_{\theta+\xi}^s$
 - ▶ same assumptions as before (MLRP, etc.)
- Social learning takes place through economic aggregates which reveal

$$m_t = (1 - \mu) \bar{F}_{\theta+\xi}^s(s_t^*) + \mu \varepsilon$$

- Assume public signal $S_t = \theta + u_t$ which capture media, statistical agencies, etc.
- No additional uncertainty, hence information evolves **identically to learning model**

◀ Return to summary of the model

◀ Return

Business Cycle Model: Households

- Households live forever, work, consume and save in capital
- Preferences

$$E \left[\sum \beta^t \log \left(C_t - \frac{L_t^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} \right) \right], \quad \sigma \geq 1, \psi \geq 0,$$

where $C_t = \left(\int_0^1 C_{jt}^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}$ is the final good

- Law of motion for the two capitals

$$K_{jt+1} = (1 - \delta) K_{jt} + I_{jt}, j = o, n$$

- Budget constraint

$$C_t + \sum_{j=o,n} I_{jt} + \frac{B_t}{P_t} = W_t L_t + \sum_{j=o,n} R_{jt} K_{jt} + \frac{1+r_{t-1}}{1+\pi_t} \frac{B_{t-1}}{P_{t-1}} + \Pi_t$$

- Retail sector:
 - ▶ buys the bundle of goods produced by entrepreneurs
 - ▶ differentiates it one-for-one without additional cost
 - ▶ subject to Calvo-style nominal rigidity → standard NK Phillips curve
- Monetary authority follows the Taylor rule

$$r_t = \phi_\pi \pi_t + \phi_y y_t$$

Calibration: Standard Parameters

Parameter	Value	Target
α	.36	Labor share
β	.99	4% annual interest rate
θ_p	.75	1 year price duration
σ	10	Markups of about 11%
ϕ_y	.125	Clarida, Gali and Gertler (2000)
ϕ_π	1.5	Clarida, Gali and Gertler (2000)
ψ	2	Frisch elasticity of labor supply
ζ	1.71	Elas. between types of K (Boddy and Gort, 1971)

Calibration: Non-Standard Parameters

Objective: target moments from the late 90s Dot com bubble

Parameter	Value	Target
ω_o	.11	Share IT capital 1991
ω_n	.26	Share IT capital 2007
λ	1/22	Duration of NASDAQ boom-bust 1995Q4-2001Q1
θ_h	1.099	SPF's highest growth forecast over 1995-2001
θ_l	.912	SPF's lowest growth forecast over 1995-2001
s_j	$N(0, .156)$	SPF's avg. dispersion in forecasts over 1995-2001
μ	15%	Fraction of noise traders
ε	Beta(2, 2)	Non-uniform distribution over [0, 1]
p_0	0.20	See below
q_0	0.15	See below

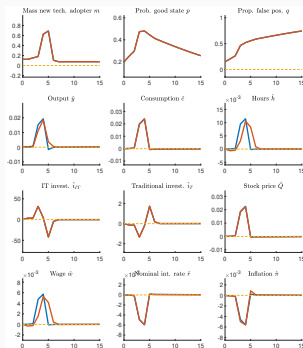
Tricky parameters:

- Noise traders μ and ε : little guidance in the literature (David, et al. 2016)
 - ▶ Sensitivity $\mu \in [0.02, 0.2]$: agents learn too fast if $\mu < 0.02$, too slowly if $\mu > 0.2$ (no quick collapse)
- p_0, q_0 : hard to tell with a single historical episode
 - ▶ The paper offers sensitivity over these two parameters

▶ Return

- Taylor rule that leads against the wind:

$$r_t = \phi_\pi \pi_t + \phi_y (y_t - \bar{y}) + \phi_i (\bar{i}_t^T - \bar{i}^T)$$



- A **leaning-against-the-wind** monetary policy:
 - ▶ Dampens fluctuations in output (welfare +0.002%)
 - ▶ But fails to improve tech adoption threshold and info collection
 - ▶ Other more directed tools (tech subsidies/taxes) more promising