Optimal Redistributive Policy in a Labor Market with Search and Endogenous Participation

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September 30, 2014

Preliminary

Abstract

We study optimal redistributive policies in a frictional model of the labor market. Ex-ante heterogeneous agents choose how much to search in a labor market characterized by a matching technology. We first derive efficiency results and provide policies to decentralize the optimal allocation. We then solve the mechanism design problem of a government with redistributive motives and limited information about agents. A large emphasis is put on the general equilibrium effects of policies on wages and job creation. We show that the optimal policy can be implemented by a non-linear income tax on workers along with an unemployment insurance program. We calibrate our model to the US economy and characterize the welfare gains from the optimal policy and its effects on output, search, wages and unemployment distribution. Our findings suggest that optimal policies often feature a generous unemployment insurance along a negative income tax that efficiently raises the participation and employment of low-income earners.

1 Introduction

Since the 1970s, means-tested transfer programs like the *Earned Income Tax Credit* (EITC) in the US, or the *Working Tax Credit* in the United Kingdom, have been developed in many countries. These programs were initially advocated as a particularly efficient way to achieve a government's redistributive objectives, while reducing the administrative cost and adverse incentives created by the overlap of many other welfare or social security programs. Such Negative Income Tax (NIT) programs have, in particular, the attractive property that they can raise work incentives and increase participation in the labor market, thereby reducing the cost of unemployment insurance programs and counteracting the adverse effects of minimum wage policies.

Negative income taxes are fairly common in the optimal taxation literature. But the standard static Mirrleesian framework treats the labor supply decision as a mere choice over leisure. Wage rates are usually exogenous, and people can seamlessly move from unemployment to employment, choosing how many hours to work.

We relax this assumption and consider the joint design of labor market policies in a search-and-matching framework for a government with redistributive motives. In such a framework, people of different skills can face different rates of unemployment and may find it more or less difficult to find a job. This unemployment risk can, arguably, be an important source of inequality if people have limited access to insurance markets. In this paper, we want to understand how a government can optimally balance efficiency and equity in the presence of search frictions and what instruments are part of the optimal policy mix. Negative income taxes can potentially have a large impact on the participation (*extensive margin*) and search intensity of agents (*intensive margin*). Is it optimal to use an NIT to make people choose to work? How should it be designed to optimally trade-off work incentives and insurance? What is the optimal design of the unemployment insurance system? Are minimum wage policies, hiring subsidies or firing costs needed for the optimal policy? We stress, in particular, the need to design these policies jointly and consider their general equilibrium effects on wages and job creation. As an example of such effects, negative income taxes can lower wages, which, in a standard framework, may lead firms to post more vacancies and reduce unemployment. By substantially alleviating the burden of the unemployment insurance program for the government, the gains of such policies can be quite large.

We study the optimal joint policy design in a static search framework along the lines of Pissarides (2000). We extend the model by introducing ex-ante heterogeneous workers and put a large emphasis on the endogenous participation and search decision of workers. The introduction of heterogeneous workers can change the efficiency properties of the model, so we consider two versions: a segmented market case, where workers of different types search on different labor market segments, and a single market case, in which they all search together on the same market. Efficiency results are derived for both specifications and we provide policies to implement these allocations. We relate these findings to well-known results in the search literature and show, in particular, that the well-known Hosios condition may not hold with heterogeneous workers.

We then consider the problem of a government with redistributive motives, facing a restricted amount of information about agents. The government therefore solves a mechanism design problem taking into account the incentives for firms and workers to take action and report more or less truthfully to the government. We show that the optimal policy can be implemented with a non-linear income tax on workers and an unemployment insurance program. Minimum wage policies and hiring subsidies are absent from the optimal policy mix.

The large number of general equilibrium effects and incentive constraints limit our ability to derive analytical expressions for the optimal tax rates. We, therefore, simulate the model and explore how optimal policies vary under different sets of parameters. We find that the optimal policy often features large unemployment benefits. The adverse effects of these benefits are counterbalanced by a negative income tax that prevents the labor market participation of the lowest-skilled to dip. The optimal tax sometimes decreases for the highest-skilled, showing a specific trade-off for the government, that would like to raise taxes but can risk lowering their search effort. Optimal policies are especially important in the single market economy as they significantly reduce the natural ineffiencies present in the model by getting closer to the efficient search intensity. The welfare gains from implementing the optimal policy are quite large, from 4 to 8%.

Related literature

This paper is related to previous literature on the optimal design of labor market institution and policy. Our approach is most closely related to Blanchard and Tirole (2008). Their paper examines the joint design of unemployment insurance and employment protection by solving a mechanism design problem in a simple model of the labor market and then providing a way to implement it using unemployment benefits, layoff and payroll taxes. Our paper follows a similar methodology, but focuses on the design of policies to induce job creation, labor market participation, and search across heterogeneous agents. Similarly, Mortensen and Pissarides (2002) investigates the effects of taxes and subsidies on labor market outcomes and characterizes the optimal policy in the labor market. Their paper restricts the set of policy instruments to a linear payroll tax, a job destruction tax and unemployment compensation. Based on a similar model, our paper improves on their approach by solving the optimal mechanism problem, i.e. by first characterizing the optimal allocation and then finding a set of policies to implement it. Our work is also related to a recent paper Golosov et al. (2011) that studies the optimal design of insurance against search risk in a model of directed search and risk-averse homogeneous agents.

Our paper also draws on optimal unemployment insurance literature as in Shavell and Weiss (1979), Wang and Williamson (1996) and Hopenhayn and Nicolini (1997). These articles mostly focus on the moral hazard problem that arise from the inability for the insurer to monitor the job search effort and job performance of the worker. These papers deliver important results on the optimal timing of benefits and their negative relationship with unemployment duration. Because these issues have already been extensively studied to a certain extent, we put the timing dimension aside and focus on the cross-sectional dimension of policies for workers of different types.

This paper is also related to the static optimal taxation literature such as Mirrlees (1971), Atkinson and Stiglitz (1980), Stiglitz (1988)) or Tuomala (1990), but rejects the assumption of a frictionless labor market. In that sense, this paper is much closely related to Hungerbühler et al. (2006) that studies optimal taxation in an imperfect labor market. This paper uses a similar search model with risk-neutral heterogeneous agents, but focuses exclusively on the redistributive aspects of taxation and its impact on job creation. Participation is exogenous, so the large impact of negative income taxes on the search dimension, through which we find large efficiency gains, is ignored. Their paper develops interesting insights on the optimal tax schedule and a number of results are derived about wages and unemployments levels in comparison with the efficient allocation.

2 Environment

We consider the problem of a government designing optimal policies in a labor market with search frictions. We first present the environment and then characterize its efficiency properties.

We build a search and matching model along the lines of Mortensen and Pissarides (1994) and Pissarides (2000), where search is random and wages are determined through Nash bargaining. We extend the standard model to allow for ex-ante heterogeneous workers and focus on two extreme specifications for the organization of the labor markets. The specific assumption we make about how different skills interact with each other are of first-order importance for the type of optimal policies that arise in this setup. In the first setup, the *segmented market* case, there is one market for each type of worker and the type is known by firms. In the second one, the *single market* case, there is a unique labor market in which all workers search.

2.1 Population and Technology

The model is static and there is a unique consumption good. The economy is populated by a continuum of mass 1 of ex-ante heterogeneous agents that differ only in their productivity level $y \geq 0$. The cumulative distribution of productivity is $G(y)$, where G is continuous and differentiable. We denote the corresponding probability density function by $g(y)$. Types are constant and agents know their own productivity. Agents are assumed risk-neutral. When unemployed, workers have some home production equal to z, independent of y. They decide whether or not to look for a job and how much intensity to put in their search. A level of effort $e \geq 0$ induces a search cost $\rho(e)$ for workers. $\rho(e)$ is positive, convex, twice-differentiable and satisfies $\rho(0) = 0$ and $\lim_{e \to 1} \rho(e) = \infty$.

There is a potentially infinite mass of homogeneous firms with free-entry. As in the Mortensen-Pissarides model, each firm can only hire a single worker. When a firm and a worker of type y are matched, the production is solely determined by the worker's productivity level and equal to y. Firms are also risk-neutral.

2.2 Labor market

In this economy, frictional unemployment arises because information about job opportunities disseminates slowly, and match creation takes time. As usual in the matching literature, this model is subject to a *congestion* externality. Agents' decisions to participate and search for a job has an adverse effect on the job finding probability of others. The more people there are on the job market, the less likely they find a partner. In our model, this externality is further amplified in the single market case by a *composition* effect due to the heterogeneity of workers. The search decisions by different types of agents have a differential impact on the economy, which requires the use of specific policies to restore efficiency.

2.2.1 Segmented market case

There is a market for each type of worker that we label by their productivity level y. Firms can therefore post type-specific vacancies for each type $v(y)$. The cost for firms to post a vacancy is $\kappa(y)$. Matches are randomly created according to an aggregate matching function $M(e(y)l(y), v(y))$, where e is the amount of search effort from workers, l the mass of workers searching and v the number of vacancies posted by firms. Following the literature, we assume that M has constant returns to scale. We define the market tightness for each market $\theta(y) \equiv v(y)/l(y)$ and denote the job filling probability per vacancy for firms $q(e,\theta) = \frac{M(el,v)}{v} = M(e/\theta, 1)$ and the job finding rate per unit of effort $f(e,\theta) = M(el,v)/el = M(1,\theta/e)$. Under this notation, a worker of type y with search effort $\tilde{e}$ finds a job with probability $\tilde{e}f(e(y),\theta(y))$.

2.2.2 Single market case

In this case, there is a unique labor market in which all types search. As in the previous case, matches are randomly created, but firms cannot direct their search towards some specific worker type. The cost per vacancy is κ. Matches are created according to the matching function $M\left(\int e(y)l(y)dy, v\right)$, with the same properties as in the previous case. We denote the market tightness by $\theta = v/\int l(y)dy$, the average effort level by $E = \int e(y)l(y)dy/\int l(y)dy$ and the total number of searching workers by $L = \int l(y)dy$. The job filling probability per vacancy is $q(E,\theta) = M(EL,v)/v = M(E/\theta,1)$ and the job finding rate per unit of effort $f(E,\theta) = M(EL,v)/EL = M(1,\theta/E)$. With this notation, a worker with search effort $\tilde{e}$ finds a job with probability $\tilde{e}f(E,\theta)$.

2.2.3 Timing

The timing is common to both cases. At the beginning of the period, workers choose their search intensity e, and firms choose how many vacancies to post. Initially, all workers start unemployed, so $l(y)$ is equal to $g(y)$. Then, matching takes place. When a firm and a worker meet, the type of the worker is revealed to the firm. The wage is determined through Nash bargaining. At the end of the period, newly created matches produce an amount equal to the worker type y, and unemployed workers produce z at home. The unemployment rate per type at the end of the period is

$$u(y) = 1 - e(y)f,$$

where f is either $f(e(y)l(y), \theta(y))$ in the segmented market case, or $f(EL, \theta)$ in the single market case.

2.3 Agents' decisions

2.3.1 Workers

At the beginning of a period, a worker of type y solves the following problem:

$$U(y) = \max_{e} -\rho(e) + efw(y) + (1 - ef)z. \tag{1}$$

He chooses how much effort e to put into search, knowing that he will find a job with probability $e \times f$. If the job is created, the worker earns the wage $w(y)$, otherwise he stays unemployed and enjoys home production z. The optimal search effort $e(y)$ can be characterized by the first-order condition

$$\rho'(e(y)) \geq f \cdot (w(y) - z),$$

with equality if $e(y) > 0$.

2.3.2 Firms

Firms post vacancies as long as the anticipated profits exceed the vacancy cost. We can write the value of an entering firm as

$$V(y) = -\kappa(y) + q(e(y), \theta(y))(y - w(y)),$$

in the segmented market case. Firms pay a vacancy cost κ, knowing that they will find a candidate at probability q. If the job is created with a type-y agent, an amount y is produced and a wage $w(y)$ is paid to the worker. Free-entry $V(y) \leq 0$ implies

$$\kappa(y) \geq q\big(e(y), \theta(y)\big)\big(y - w(y)\big). \tag{2}$$

In the single market case, a similar, but unique, free-entry condition applies:

$$\kappa \geq q(E,\theta) \int \frac{e(y)l(y)}{EL}\big(y - w(y)\big)dy. \tag{3}$$

These inequalities bind if $\theta > 0$.

2.3.3 Wages

When a worker of type y and a firm meet, the surplus is split according to a standard Nash bargaining procedure:

$$w(y) = \underset{w}{\operatorname{argmax}}[w-z]^{\gamma}[y-w]^{1-\gamma}, \tag{4}$$

where γ is the bargaining power of the worker.

2.4 Equilibrium

We now define the notion of a competitive equilibrium in both economies. For the segmented market case:

Definition 1. *A competitive equilibrium of the segmented market economy is a market tightness $\theta(y)$, a search effort schedule $e(y)$, a vacancy posting schedule $v(y)$, and a wage schedule $w(y)$ such that:*

1. *Wage $w(y)$ solves the Nash bargaining procedure* (4)*,*
2. *Search effort $e(y)$ solves the worker's problem* (1)*,*
3. *Free-entry condition* (2) *is satisfied.*

Similarly, for the single market case:

Definition 2. *A competitive equilibrium of the single market economy is a market tightness θ, a number of vacancies v, a mean search intensity E, a search effort schedule $e(y)$, and a wage schedule $w(y)$ such that:*

1. *Wage $w(y)$ solves the Nash bargaining procedure* (4)*,*
2. *Search effort $e(y)$ solves the worker's problem* (1)*,*
3. *Free-entry condition* (3) *is satisfied,*
4. *Mean search intensity E satisfies $E = \int e(y)l(y)dy / \int l(y)dy$.*

3 Efficiency

We now analyze the efficiency properties of the model and provide, if available, policies to decentralize the optimal allocation.

3.1 Segmented market case

In the segmented market case, workers only search in their appropriate market segment and there are no interactions between skill groups. The only source of inefficiency is a standard congestion externality for each skill: when a worker increases his search effort, the probability at which other workers find jobs decreases, while it becomes easier for firms to recruit candidates. This case is therefore close to the standard Mortensen-Pissarides model and we show that an Hosios condition is sufficient to implement the optimal allocation.

We consider the constrained-efficient allocation of a planner subject to the search and matching technology. The planner is allowed to choose how many vacancies to post on each labor market segment, as well as the search intensity for each skill group, but cannot move workers directly. The social objective is to maximize:

$$\begin{aligned} \max_{\theta(y),e(y),u(y)} & \int [(1-u(y))y + u(y)z - \rho(e(y)) - \kappa(y)\theta(y)]g(y)\mathrm{d}y \\ \text{s.t} \quad & u(y) = 1 - e(y)f(e(y),\theta(y)) \end{aligned} \tag{5}$$

which is the sum of the total production in the economy net of search and vacancy costs by workers and firms.

Proposition 1 (Hosios condition)**.** *The segmented market economy is not efficient in general, but the constrained efficient allocation can be implemented by setting the bargaining power equal to the elasticity of the matching function:*

$$\gamma(y) = \frac{\partial M}{\partial el}\frac{e(y)l(y)}{M(e(y)l(y),v(y))}.$$

This proposition extends the well-known result provided by Hosios (1990) to a model with heterogeneous workers and segmented markets. When the bargaining power of workers is equalized to the elasticity of the matching function with respect to the total worker search effort, then the search externality is fully internalized by agents, and their individual incentives are aligned with the social values. This result does not come as a surprise. Given that workers search on independent markets, it seems natural that an Hosios condition holds on each market. Notice however that the bargaining power may in general depend on the type y. A specific case arises when the matching function is of the Cobb-Douglas form, in which case it is optimal to set the bargaining power constant across workers.

3.2 Single market case

We now turn to the single market economy. On top of being subject to the congestion externality, the composition of the labor force now matters. Indeed, on a single market segment, workers of all types face the same job finding probability. Yet, their social values may substantially differ. As a result, we might expect the level of search in the competitive economy to be at odds with the optimal allocation: unskilled workers will search too much, while skilled workers will not search enough. We show in this section that this is, indeed, what happens and that a standard Hosios condition is not sufficient to implement the optimal allocation. A form of taxation and wage subsidy is needed to restore efficiency.

Analogously to the segmented market case, the social planner in the constrained efficient allocation maximizes the following welfare function subject to the matching frictions:

$$\max_{\theta,e(y),u(y)} \int [(1-u(y))y + u(y)z - \rho(e(y))]g(y)\mathrm{d}y - \kappa\theta \tag{6}$$

$$\text{s.t} \quad \begin{cases} u(y) = 1 - e(y)f(E,\theta) \\ E = \int e(y)l(y)dy / \int l(y)dy \end{cases} .$$

Proposition 2. *The competitive allocation of the single market economy is not efficient in general, and a standard Hosios condition is not sufficient to implement the constrained efficient allocation. However, under the Hosios condition $\gamma = \frac{\partial M}{\partial EL}\frac{EL}{M(EL,v)}$, a uniform tax on workers T, a linear wage subsidy s and a vacancy subsidy S_v implement the constrained optimum.*

Proposition 2 tells us that the Hosios condition is no longer enough to let the competitive equilibrium implement the constrained-efficient allocation. This proposition is reminiscent of a result from Shimer and Smith (2001) that similarly established that a type-specific subsidy on search intensity could restore efficiency in a model with ex-ante heterogeneous agents and constant returns to scale matching technology.

This inefficiency result comes from a composition effect in the congestion externality. Workers of different skills have different social values, and, yet, they impose the same crowding effect to each other. The welfare losses come from the fact that low-skilled workers search too much: the social benefits of them having a job is rather low, while they prevent higher skilled workers to get jobs. As a result, the planner would like to make low-skilled workers search less and high-skilled workers search more. It can do so by using the uniform tax T to prevent entry at the bottom of the skill distribution and the wage subsidy s to raise incentives to search for high-skilled workers. The vacancy subsidy is only used to restore the efficient level of job creation. To illustrate this, let us look at the optimal search effort condition. To simplify, let us specialize the matching function to the Cobb-Douglas function $M(EL,v) = (EL)^{\alpha}v^{1-\alpha}$. The optimal search effort in the planner's problem is given by

$$\rho'(e(y)) = \underbrace{f(E,\theta)(y-z)}_{\text{individual returns to search}} - \underbrace{(1-\alpha)f(E,\theta)(\bar{y}-z)}_{\text{across-skill crowding effect}},$$

where $\bar{y}$ is the effort-weighted average skill on the labor market, and in the decentralized equilibrium without distortionary taxes:

$$\rho'(e(y)) = \gamma f(E,\theta)(y-z).$$

It is immediate to see that any bargaining power less than one for the worker will have trouble making them internalize their full social value $y-z$ and will thus induce a lower search effort. This does not happen in the segmented market case, with $\bar{y}=y$, so that:

$$\rho'(e) = f(e,\theta)(y-z) - \underbrace{(1-\alpha)f(e,\theta)(y-z)}_{\text{per-skill crowding effect}} = \alpha f(e,\theta)(y-z),$$

where it is easy to see that setting the bargaining power $\gamma = \alpha$ is enough to implement the social optimum.

To illustrate this result further, we simulate the economy under some set of parameters and compute the efficient search schedule together with that of a competitive equilibrium under the Hosios condition. Figure 1 shows the resulting schedules.

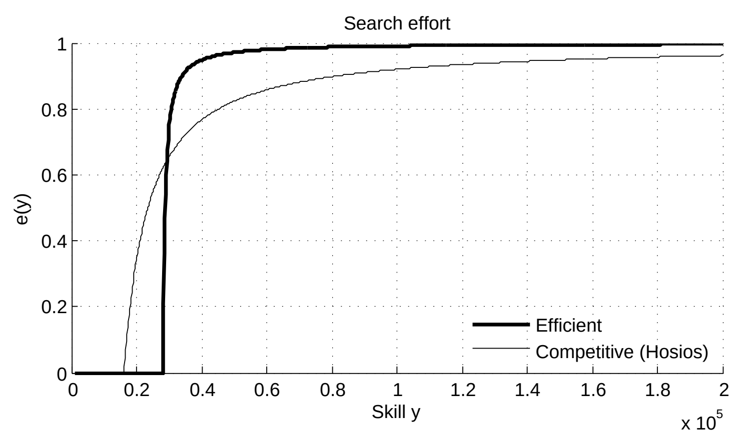


Figure 1: Efficient and competitive search schedules in the single market case

cumulative

Figure 1 tells us that the two allocations can greatly differ and confirms that the general view that the Hosios condition is sufficient to restore efficiency in search-and-matching models may not hold in an economy with heterogeneous agents. In particular, in the single market case, participation on the extensive margin is much lower in the efficient allocation: workers at the bottom of the distribution participate less, while the search schedule is initially much steeper for the first entering skill group.

4 Optimal Redistributive Policy

We have studied, in the previous section, the efficiency properties of the model under its two extreme specifications in order to make clear the key inefficiencies of the model. We now explore optimal redistributive taxation. In particular, our objective is to understand how the trade-off between efficiency and equity can be resolved. What type of policies arise under reasonable assumptions on the government's information set? Is a negative income tax optimal and does it play an active role in raising participation and employment, while fulfilling some of the redistributive goals? What is the optimal level of unemployment insurance? What are the general equilibrium effects of these policies?

4.1 Government's objective

There is a government with the following social welfare function:

$$\int \Big[(1-u(y))\Phi\big(c_e(y)-\rho(e(y))\big)+u(y)\Phi\big(c_u(y)-\rho(e(y))\big)\Big]g(y)\mathrm{d}y, \tag{7}$$

where $c_e(y)$ and $c_u(y)$ are the consumption levels of type-y agents while employed and unemployed. Φ describes the government's preference for redistribution. We assume that $\Phi' > 0$ and $\Phi'' \leq 0$. The higher the degree of concavity of Φ, the more the government will favor redistributive policies.[1] As in the previous section, search frictions still constrain the allocation, and the government cannot move workers freely from unemployment to employment.

4.2 Information set and mechanism design

Under the above preferences, the optimal allocation for the government is to choose the efficient levels of search effort and vacancy posting, but set the utility levels equal across types and across employent statuses (employed vs. unemployed). Such an allocation may not be implementable in practice: it is unlikely that the government can control the amount of search by agents, as well as whether firms are actually searching for workers or simply pretending to have vacancies posted to receive subsidies (if any). Also, most labor market policies in reality are based on the wage (or income), which is the only information available to the government.

To capture these limits to the government's actions, we assume that the worker's search effort is unobservable. His employment status is, however, observable and the government can offer two distinct sets of transfers when employed or unemployed. On the firm

[1] We maintain the assumption that agents are risk-neutral for convenience. Risk-aversion can be introduced in the model, but can significantly complicate the Nash bargaining. This does not affect the main points we wish to draw in this version of paper, so we leave this for future versions.

side, similar issues may arise with the possibility that firms post vacancies without actually searching for workers. To prevent such complications, we assume directly that only firms with filled vacancies are observable. Therefore, only transfer to these firms are available and the free-entry condition must be satisfied. Under these assumptions, the types of workers and worker-firm pairs are unknown to the government. In particular, unemployed workers of different skills are indistinguishable. To this very limited information set, we make the additional assumption that the government can observe the outcome of the Nash bargaining process for matched worker-firm pairs. This provides additional information to the government and allows it to design transfers based on that outcome. We thus focus on mechanisms in which the matched worker-firm pairs decide on a type to declare to the government through Nash bargaining. We further assume that the bargaining power γ is not a policy instrument, but a structural parameter of the economy that cannot be chosen directly by the government.

The government offers the following set of transfers: c_u, a transfer to the unemployed, $c_e(\tilde{y})$ a transfer to employed workers and $T_f(\tilde{y})$ a transfer to firms having jointly declared a type $\tilde{y}$. When a worker and a firm meet, they negotiate over the type to declare to the government:

$$\max_{\tilde{y}}[c_e(\tilde{y}) - c_u - z]^{\gamma}[y + T_f(\tilde{y})]^{1-\gamma}.$$

Assuming that the government can observe all transfers between the firm and the worker, there is no loss of generality in assuming that these within-pair transfers are 0.[2] Using the revelation principle, we focus on truth-telling mechanisms.

Proposition 3 provides an implementation result that enables us to focus directly on an optimal taxation problem.

Proposition 3. *(a) Any truth-telling mechanism $\{c_e(\cdot), T_f(\cdot), c_u\}$ can be implemented in a competitive equilibrium by a non-linear income tax on workers $\tau(w)$ and uniform unemployment insurance b.*

(b) The corresponding equilibrium schedules $w(y)$ and $w(y) - \tau(w(y))$ are increasing functions of skill y.

Thanks to proposition 3, we know that we can directly maximize over the the two policy instruments $\tau(\cdot)$ and b. A single-crossing property tells us, on top of the implementation result, that the resulting wage and wage net of taxes will be increasing with the skill. Notice that this proposition establishes only one implementation among, possibly, many others. No hiring subsidies to firms are needed, in theory, for this implementation to hold. We

[2] If the worker and the firm were willing to transfer some ressources (through a wage for example), it is always possible to rewrite another set of government transfers such that this wage is 0.

have not, however, put any formal restriction on the sign of the wage. Under this implementation, wages could in principle become negative, as a way to subsidize firms. This is not necessarily an unrealistic result and we can always rewrite another equivalent set of policies with a uniform hiring subsidy to firms, but positive wages (see appendix B).

4.3 Optimal Taxation

We now state the full optimal taxation problem. Common to both specifications of the labor market, the government solves the following problem:

$$\max_{\substack{w(\cdot),e(\cdot),\theta(\cdot),\\ u(\cdot),\tau(\cdot),b}} \int \Big[(1-u(y))\Phi\big(w(y)-\tau(w(y))-\rho(e(y))\big)+u(y)\Phi\big(b+z-\rho(e(y))\big)\Big]g(y)\mathrm{d}y$$

$$\begin{aligned}
\text{s.t}\quad & u(y)=1-e(y)f && \text{(Unemployment)}\\
& w(y)=\operatorname*{argmax}_w [w-\tau(w)-b-z]^{\gamma}[y-w]^{1-\gamma}, \quad \forall y && \text{(Nash)}\\
& \rho'(e(y))=f\cdot\big(w(y)-\tau(w(y))-b-z\big), \quad \forall y && \text{(Effort)}\\
& \int \big[(1-u(y))\tau(w(y))-u(y)b\big]g(y)dy\geq 0 && \text{(Budget)}\\
& +\ \text{economy-specific constraints,}
\end{aligned}$$

where f should be understood as $f(e(y),\theta(y))$ in the segmented market case or $f(E,\theta)$ in the single market case. Constraint (Nash) captures the incentive constraint of worker-firm pairs bargaining over the wage. Constraint (Effort) is the incentive constraint corresponding to the hidden search effort chosen by workers. Equation (Budget) is the government's budget constraint.

We must also include the resource constraint (Resource) and the free-entry condition (Free-entry), since the government cannot control job creation directly. In the segmented market economy, these constraints can be written:

$$\begin{aligned}
&\int \big[(1-u(y))(w(y)-\tau(w(y)))+u(y)(b+z)+\kappa(y)\theta(y)\big]g(y)dy\\
&\quad=\int \big[(1-u(y))y+u(y)z\big]g(y)dy && \text{(Resource)}\\
&\kappa(y)=q(e(y),\theta(y))(y-w(y)), \quad \forall y. && \text{(Free-entry)}
\end{aligned}$$

The corresponding constraints in the single market economy become:

$$\begin{aligned}
&\int \big[(1-u(y))(w(y)-\tau(w(y)))+u(y)(b+z)\big]g(y)dy+\kappa\theta\\
&\quad=\int \big[(1-u(y))y+u(y)z\big]g(y)dy && \text{(Resource)}\\
&\kappa=q(E,\theta)\int (y-w(y))e(y)g(y)dy\Big/\int e(y)g(y)dy. && \text{(Free-entry)}
\end{aligned}$$

4.4 Solution method

It is difficult to derive analytical results for the optimal taxation problem because of the large number of incentive constraints and general equilibrium effects present the model. We, therefore, solve the problem numerically and proceed to a number of comparative statics to highlight the mechanisms at play in the economy. Before doing so, we turn the problem into a form that can be solved using standard numerical methods.

The key issue with the optimal taxation problem, as stated currently, is the Nash incentive constraint

$$w(y) = \underset{w}{\operatorname{argmax}} [w - \tau(w) - b - z]^{\gamma} [y - w]^{1-\gamma}, \quad \forall y,$$

which is a collection of an infinite number of non-linear inequality constraints. To deal with this issue, we use a first-order approach and restrict the space of policies to differentiable functions. We derive the first-order conditions of this maximization problem:

$$\frac{\gamma(1 - \tau'(w(y)))}{w(y) - \tau(w(y)) - b - z} = \frac{1 - \gamma}{y - w(y)}, \quad \forall y. \tag{8}$$

This condition can be further simplified if we define the Nash product

$$\Sigma(y) \equiv \max_{w} [w - \tau(w) - b - z]^{\gamma} [y - w]^{1-\gamma},$$

then the first-order condition (8) is also equivalent to the envelope condition:

$$\frac{\Sigma'(y)}{\Sigma(y)} = \frac{1 - \gamma}{y - w(y)}. \tag{9}$$

The first-order condition is necessary but not necessarily sufficient. We thus verify in our simulations that the solution is indeed a maximum (a single-crossing property can also be used for sufficiency).

Substituting constraint (Nash) with equation (9) enables us to transform the taxation problem in an optimal control problem using $\Sigma(y)$ as a state variable and $w(y)$ as control. For the segmented market economy, we solve the following problem:

$$\max_{\substack{w(\cdot),\Sigma(\cdot),e(\cdot),\\ \theta(\cdot),u(\cdot),b}} \int \Big[(1 - u(y))\Phi\big(w(y) - \tau(w(y)) - \rho(e(y))\big) + u(y)\Phi\big(b + z - \rho(e(y))\big) \Big] g(y) \mathrm{d}y$$

$$\text{s.t} \quad u(y) = 1 - e(y) f(e(y), \theta(y)) \tag{Unemployment}$$

$$\frac{\Sigma'(y)}{\Sigma(y)} = \frac{1 - \gamma}{y - w(y)} \tag{IC}$$

$$\rho'(e(y)) = f \cdot \big(w(y) - \tau(w(y)) - b - z\big) \tag{Effort}$$

$$\int \big[(1 - u(y))\tau(w(y)) - u(y) b\big] g(y) dy \geq 0 \tag{Budget}$$

$$\kappa(y) = q(e(y), \theta(y))(y - w(y)), \quad \forall y. \tag{Free-entry}$$

Constraints (Unemployment), (Effort) and (Free-entry) can be directly substituted into the objective function. These manipulations finally give us an optimal control problem with the unique linear inequality constraint (Budget), so standard Hamiltonian techniques apply.[3] Notice that we have dropped the resource constraint because of its redundancy with the government's budget constaint and free-entry conditions. The full statement of the optimal control problem in the single market economy can be found in appendix A.

5 Simulations

In this section, we calibrate the two specifications of the model and compute the optimal policies with different degrees of preference for redistribution. We highlight characteristics of the optimal policy and explain how the various policy instruments affect the allocation. We then compare the allocations with the efficient one and the calibrated US economy to produce welfare comparisons.

5.1 Calibration

We calibrate the model to match some particular features of the US economy. The time period is set to a year. The skill distribution is assumed to follow a log-normal distribution of parameters (μ_y, σ_y^2). To calibrate it, we construct a measure of skills using the Current Population Survey (CPS) in the year 2000. We regress weekly earnings[4] on several characteristics: age, occupation, sex, education, race, industry and state. Our measure of skill is constructed for each worker using the projection of his earnings on his individual characteristics (thus excluding industry and state). Using this measure, we obtain a distribution of before-tax earnings and construct a distribution of unemployment rates per skill.

We calibrate the skill distribution on the empirical distribution of before-tax earnings. We use a measure of effective income tax rates in the US in 2000 from Guner et al. (2011), whose preferred estimate for the average tax rates is:

$$t(\tilde{y}) = \alpha + \beta \log(\tilde{y}), \quad \alpha = 0.1127 \text{ and } \beta = 0.04,$$

with $\tilde{y}$ a multiple of mean household income. It is common in the search litterature to set the bargaining power of workers to values between 0.5 and 0.7, so we choose $\gamma = 0.5$. Given

[3]The key is that the tax $\tau(w(y))$ can be recovered from the state variable $\Sigma(y)$ using

$$\tau(w(y)) = w(y) - b - z - \left[\frac{\Sigma(y)}{(y - w(y))^{1-\gamma}}\right]^{\frac{1}{\gamma}}.$$

[4]We use weekly earnings for workers between the age of 16 and 65. Observations under \$4/hr and over \$100/hr are dropped. We rescale this measure of weekly earnings to obtain annual earnings.

this bargaining power and tax rate, we can compute the before-tax earnings distribution in the model, common to both specifications of the labor market. The log-normal distribution parameters (μ_y, σ_y^2) are calibrated to provide the best fit for this distribution. Our preferred parameters give us an almost perfect fit, as figure 2 shows.

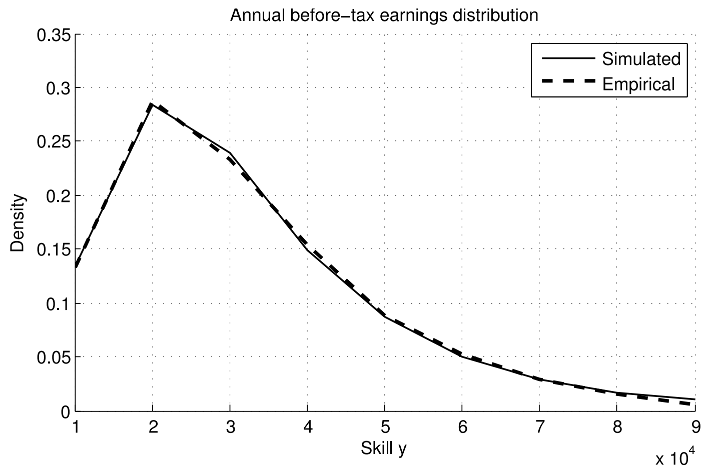


Figure 2: Fit of the empirical and simulated distributions of before-tax earnings

We are now left to calibrate labor market variables. We use the following functional form for the matching function:

$$M(el, v) = \frac{el \cdot v}{((el)^{\alpha} + v^{\alpha})^{1/\alpha}},$$

so that $f(e,\theta) = \frac{\theta}{e}\left[1+(\frac{\theta}{e})^{\alpha}\right]^{-1/\alpha}$ and $q(e,\theta) = \left[1+(\frac{\theta}{e})^{\alpha}\right]^{-1/\alpha}$. This form of matching function is often used in the search literature and is quite convenient as it yields job finding and job filling probabilities bounded between 0 and 1. For the search cost function, we choose

$$\rho(e) = A \log\left(\frac{1}{1-e}\right),$$

which provides a nice fit for our calibration strategy and produces effort levels bounded below 1.

5.1.1 Single market case

We start by calibrating the single market case. To set the vacancy cost κ, we use estimates from Silva and Toledo (2009) who report, using survey data, that interviewing and training costs for workers amount to 34.4% (3.4% and 31%) of the average annual worker salary. Using this estimate, we set κ equal to 34.4% of the average annual salary in the economy. Calibrating b and z raises a number of issues. First, only their sum matters for the economy, unless we had data on the government's budget. We therefore focus on their sum. Second,

$b+z$ is constant across workers, because the model is static and the government cannot use past wages to vary b across skills. This dimension of the model is slightly too stylized to be able to match the data, and there is unfortunately no fully satisfactory way to calibrate these parameters. Instead of using data on unemployment insurance, we choose to calibrate $b+z$ to match the first entering skill in the economy, also corresponding to the minimum wage. The minimum wage in the model is such that $\underline{w}-\tau(\underline{w})-b-z=0$. The federal minimum wage in 2000 was $5.15/hr. Transforming this into an annual measure, we set $b+z$ accordingly. The only parameter left to calibrate is the search cost parameter A. We choose A so as to minimize the distance between the empirical distribution of unemployment rate we constructed using our CPS data and the simulated one. The left panel of figure 3 shows the fit with our best estimate.

5.1.2 Segmented market case

To calibrate this version of the model, we use the same parameter estimates as in the single market case, except for the vacancy costs. This version of the model is, indeed, unable to match the distribution of unemployment unless these costs vary between labor markets. We parameterize these costs as follows:

$$\kappa(y)=\kappa_0 y^{\kappa_1}.$$

Again, we calibrate the parameters κ_0 and κ_1 to minimize the distance between the empirical unemployment distribution and the simulated one. The right panel of figure 3 shows the fit in the segmented market case. Table 1 summarizes the values of our parameters in both specifications of the model.

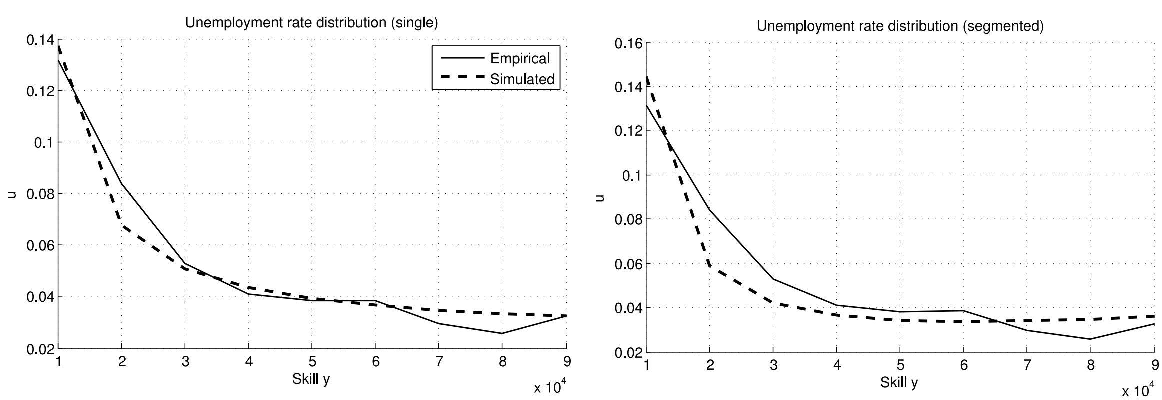


Figure 3: Fit of the empirical and simulated distributions of unemployment

Parameter	Description	Value
μ_y	Mean parameter of skill distribution	10.96
σ_y^2	Variance parameter of skill distribution	0.565
α	Matching function parameter	3.176
$b+z$	Value of unemployment	\$8,907
A	Search cost parameter	579.3
κ	Vacancy cost (single)	\$12,679
κ_0	Vacancy cost (segmented)	8.01e-3
κ_1	Vacancy cost (segmented)	1.274

Table 1: Calibrated parameters

5.2 Economic forces at work

Before presenting the results of the simulations, we take some time to highlight the different effects of the optimal policy instruments in the model.

Income tax A positive shift in τ leads workers to negotiate higher wage to share the additional tax burden with the firm. As a result, the worker and firm surpluses are both reduced. A first consequence on the worker side is a lower search effort. On the firm side, lower profits and lower search from workers leads to less vacancy posting and an unambiguously higher level of unemployment. The initial increase in tax revenue for the government are thus reduced through a lower tax base and additional unemployment benefits to pay. A point increase in τ' tends to decrease the wage at that point, lowering search, but potentially raising firm profits and lowering unemployment.

Unemployment benefits Unemployment benefits are important in this model because agents do not have access to insurance markets. Because these benefits are the only way to insure unemployed workers, a government with redistributive motives will in general choose a generous level of compensation. These benefits are, however, strongly counterbalanced by some serious adverse effects. First, raising unemployment benefits reduces participation as equation (Effort) shows. This increases the burden to the government, which must raise taxes to finance this policy. A second effect is that it raises the outside value of workers, who will demand higher wages. Higher wages lead to lower profits for firms and less vacancy posting. Unemployment increases further and the government must find other sources of funding.

5.3 Optimal policy in the segmented market economy

We compute the optimal policy for the welfare function $\Phi(x) = x^{\varphi}$ and different values of $\varphi = 1/2$ and $1/4$. $1/4$ is more concave, so the government has a stronger preference for redistribution. Figure 4 presents the results and compares some features of the allocation to the calibrated version of the US economy and the constrained-efficient allocation. Table 5.3 reports a number of important variables across the different allocations. Panel (a) presents how gross wages vary across types. We can see, first, that wages are higher under the optimal policy in both case, $\varphi = 1/2,\ 1/4$, than in the calibrated version.

Panel (b) shows the net wages, also equal to the consumption of the employed. Interestingly, consumption is quite similar across allocations for the bottom half of the population. It, however, increases much more under the optimal policy with the lower taste for redistribution. This seems a mean for the government to increase the search effort of the highest skilled in order to raise more tax revenue.

Panel (c) shows the optimal tax as a function of income. It confirms the intuition that negative income taxes seem particularly important both to redistribute and raise labor market participation for the low-skilled. The optimal tax starts negative, then becomes positive until it starts decreasing for the highest skilled. This shows a very specific trade-off for the government: it would like to tax these workers, but doing so would reduce their search and employment by too much, thereby decreasing tax revenues. Therefore, taxes remain positive but decrease slightly as productivity increases. Taxes decrease even more as φ increases: more weight is put on the high-skilled. Notice also that marginal taxes are not 0 at the top of the distribution. This is because taxes are a tool to split surplus between firms and workers and marginal taxes, in particular, affect the relative bargaining powers of workers and firms. As a result, the standard intuition that marginal tax rates should be 0 do not apply here as in the rest of the optimal taxation literature. Panel (d) displays the search effort schedule. Surprisingly, participation is lower under the optimal policies than in the calibrated and efficient allocations: the first skill that enters the labor market is much higher. This results from the adverse effects of the unemployment insurance. Since UI benefits is the only resource of unemployed individuals on top of their home production, the government is willing to give quite generous transfers. As we said in the preceding section, such a policy has very negative consequences on participation and search intensity. This does not mean that the NIT fails, but, on the contrary, that an NIT is strongly needed to counteract these negative effects.

Panel (e) presents the distribution of after-tax wages. It should be noted that the distribution shifts to the right under the optimal policies, even more so with a large φ. People enjoy more consumption, and the distribution seems to have a smaller tail.

Panel (f) presents the distribution of unemployment rates per skill. Quite surprisingly, the efficient allocation requires a much higher unemployment rate. This result comes from

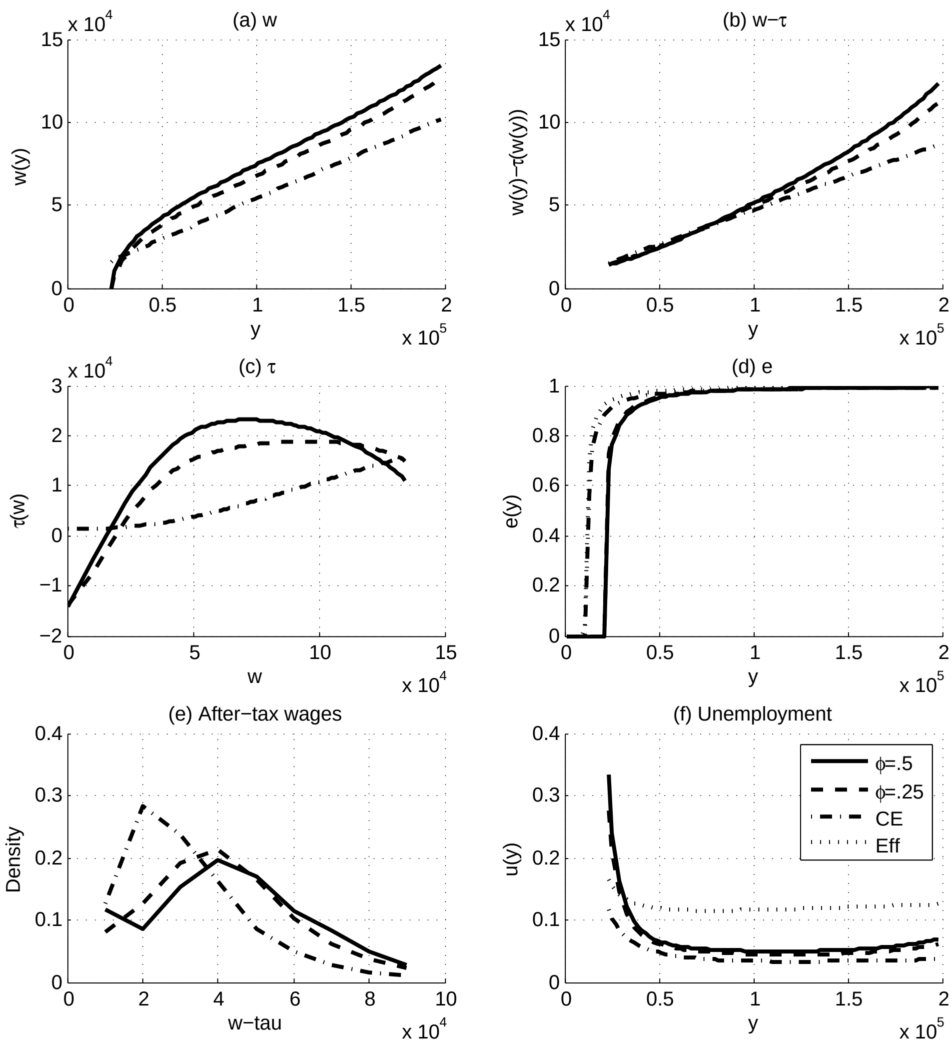


Figure 4: Optimal policies, calibrated and efficient allocations in the segmented market economy

	b+z	H	Welfare
Calibrated	8,907	0	1
$\varphi = 1/2$	12,407	11,996	1.059
$\varphi = 1/4$	11,907	7,751	1.037

Notes: Welfare is computed in consumption equivalent compared to the calibrated case.

Table 2: Unemployment benefits, hiring subsidies and welfare in the segmented case

the fact that $\gamma = 0.5$ in our calibration, much less than the efficient one that averages around 0.8. As a result, firms post too many vacancies on most markets. The unemployment rate is larger under the optimal policy and is closer to the efficient level than the calibrated one. This results mostly from the fact that gross wages are much higher and firms' profits lower. Vacancy posting is thus smaller, as required in the efficient case.

Finally, table 5.3 shows that hiring subsidies are needed under the optimal policy in both cases to sustain a sufficient level of vacancy posting (although higher than in the efficient case). Welfare increases by 4-6% in consumption equivalent, which is quite large and is coming mostly from the higher unemployment benefits.

5.4 Optimal policy in the single market economy

Figure 5 show the same optimization results in the single market economy. Table 5.4 reports welfare comparisons and the level of subsidies and unemployment benefits.

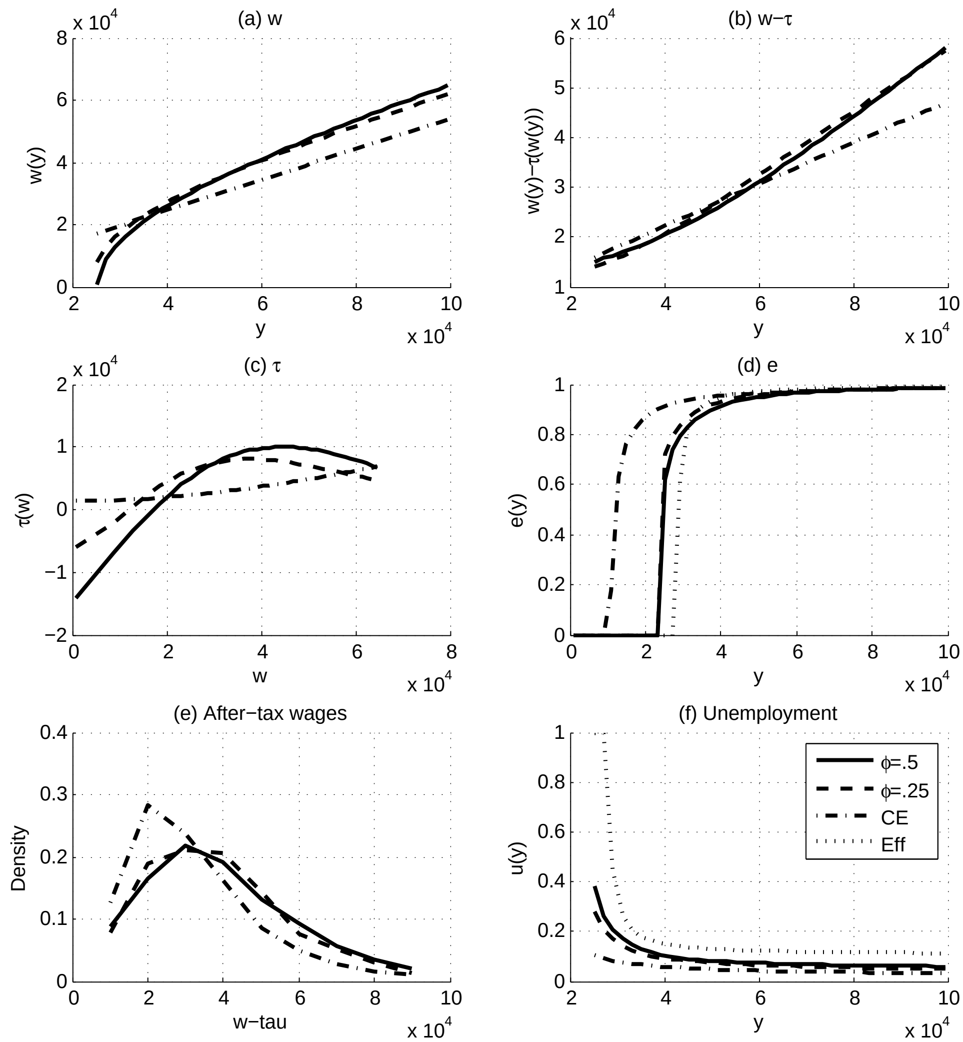
	b+z	H	Welfare
Calibrated	8,907	0	1
$\varphi = 1/2$	13,507	0	1.084
$\varphi = 1/4$	11,907	0	1.051

Notes: Welfare is computed in consumption equivalent compared to the calibrated case.

Table 3: Unemployment benefits, hiring subsidies and welfare in the single market case

Panels (a)-(c) present very similar pictures to the segmented market case. It should be noted that, again, the optimal tax is very negative at the bottom of the skill distribution, rises, then decreases again for the highest skilled. Negative income taxes therefore seem quite robust, but more sensitivity analysis is needed.

Panel (d) shows, however, a very different picture. In this case, as we explained in part 3, it is optimal to prevent the lowest skilled worker to enter the labor market because of the negative impact that they have on the job finding rates of higher skilled workers. Notice the efficient search effort has the lowest participation for people at the lower end



Notes: CE is the calibrated competitive equilibrium, Eff is the constrained-efficient allocation.

Figure 5: Unemployment benefits, hiring subsidies and welfare in the single market case

of the skill distribution. This is quite good news for the government, as it can push up the unemployment benefits without sacrificing efficiency. Notice, however, that the optimal unemployment benefits may not necessarily be much higher than in the segmented market case, because total production can be lower in this economy. The search effort under the optimal policy is much closer to the efficient one than in the calibrated economy, so these policies do a good job at correcting inefficiencies. A similar picture is shown in panel (f) as the optimal unemployment rates get closer to the efficient one than the calibrated economy.

Table 5.4 shows that no hiring subsidies are needed in the single market economy, showing that there is no need to correct job creation further. Welfare gains in consumption equivalent are quite large in this case, from 5 to 8%, probably because the optimal policies do a good job at correcting the inefficiencies present in that version of the model.

6 Conclusion and Further Extensions

Possible extensions:

- Introduce dynamics, which will, in particular, enable us to make unemployment insurance depend on past wages.
- Introduce risk-aversion.
- Possibly consider directed/competitive search.
- Enrich further comparative statics and sensitivity analysis.
- Derive analytical expressions for optimal tax rates, or at least show the impact of average/marginal tax rates on search effort, wages and job creation.

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Appendices

A Optimal control problem in the single market case

In the single market economy, the optimal control problem can be stated as follows:

$$\max_{\substack{w(\cdot),\Sigma(\cdot),e(\cdot),\\ \theta(\cdot),u(\cdot),b}} \int \Big[(1-u(y))\Phi\big(w(y)-\tau(w(y))-\rho(e(y))\big)+u(y)\Phi\big(b+z-\rho(e(y))\big)\Big]g(y)\mathrm{d}y$$

$$\begin{aligned}
\text{s.t}\quad & u(y)=1-e(y)f(E,\theta) && \text{(Unemployment)}\\
& \frac{\Sigma'(y)}{\Sigma(y)}=\frac{1-\gamma}{y-w(y)} && \text{(IC)}\\
& \rho'(e(y))=f(E,\theta)\cdot\big(w(y)-\tau(w(y))-b-z\big) && \text{(Effort)}\\
& \int\big[(1-u(y))\tau(w(y))-u(y)b\big]g(y)dy\geq 0 && \text{(Budget)}\\
& \int\big[(1-u(y))(w(y)-\tau(w(y)))+u(y)(b+z)\big]g(y)dy+\kappa\theta\\
& \quad=\int\big[(1-u(y))y+u(y)z\big]g(y)dy && \text{(Resource)}
\end{aligned}$$

We use the resource constraint instead of the free-entry condition mostly for convenience in our simulations.

B Proofs

The proofs of the propositions in the main text can be found in this section.

Proof of prop.1:
Let us derive the first-order conditions of the planner's problem. Write the Lagrangian

$$\begin{aligned}
\mathcal{L}=\int\Big[&(1-u(y))y+u(y)z-\rho(e(y))-\kappa(y)\theta(y)\\
&+\mu(y)\big(u(y)-1+e(y)f(e(y),\theta(y))\big)\Big]g(y)\mathrm{d}y,
\end{aligned}$$

where $\mu(y)$ is the Lagrange multiplier on the constraint. The optimality conditions are:

$$\frac{\partial\mathcal{L}}{\partial u(y)}=-y+z+\mu(y)=0 \tag{10}$$

$$\frac{\partial\mathcal{L}}{\partial\theta(y)}=-\kappa(y)+\mu(y)e(y)\frac{\partial f}{\partial\theta(y)}=0 \tag{11}$$

$$\frac{\partial\mathcal{L}}{\partial e(y)}=-\rho'(e(y))+\mu(y)\Bigg(\underbrace{f(e(y),\theta(y))}_{\text{direct effect}}+\underbrace{e(y)\frac{\partial f}{\partial e(y)}}_{\text{per skill crowding effect}}\Bigg)=0. \tag{12}$$

Given that the matching function is homogeneous of degree 1, we have that $M(el,v) = elM_{el} + vM_v$. Therefore:

$$
\begin{aligned}
e(y)\frac{\partial}{\partial\theta(y)}f(e(y),\theta(y)) &= M_v(1,\theta(y)/e(y)) = M_v(e(y)l(y),v(y)) \\
f(e(y),\theta(y)) + e(y)\frac{\partial f}{\partial e(y)} &= M_{el}(1,\theta(y)/e(y)) = M_{el}(e(y)l(y),v(y)).
\end{aligned}
$$

The first-order equations combine to yield the following optimality conditions:

$$
\begin{cases}
\kappa(y) = M_v(e(y)l(y),v(y))(y-z) \\
\rho'(e(y)) = M_{el}(e(y)l(y),v(y))(y-z)
\end{cases}
\tag{13}
$$

Compare to the equilibrium conditions of the competitive economy:

$$
\begin{cases}
\kappa(y) = q(e(y),\theta(y))(1-\gamma(y))(y-z) \\
\rho'(e(y)) = f(e(y),\theta(y))\gamma(y)(y-z)
\end{cases}
\tag{14}
$$

where we allow the bargaining power to depend on the type y. If the planner can choose the bargaining power such that

$$
\gamma(y) = \frac{M_{el}(1,\theta(y)/e(y))}{f(e(y),\theta(y))} = el\frac{M_{el}(el,v)}{M(el,v)},
$$

then the competitive allocation implements the constrained efficient allocation. Indeed, it is easy to verify that $f(e(y),\theta(y))\gamma(y) = M_{el}(e(y)l(y),v(y))$ and

$$
q(e(y),\theta(y))(1-\gamma(y)) = \frac{M}{v}\frac{(M-elM_{el})}{M} = M_v(e(y)l(y),v(y)).
$$

□

Proof of prop.2:
Write the Lagrangian of the planner's problem:

$$
\mathcal{L} = \int\Big[(1-u(y))y + u(y)z - \rho(e(y)) + \mu(y)\big(u(y) - 1 + e(y)f(E,\theta)\big)\Big]g(y)\mathrm{d}y - \kappa\theta,
$$

where $\mu(y)$ is the multiplier on the constraint for the unemployment level. The first-order conditions are:

$$
\frac{\partial\mathcal{L}}{\partial u(y)} = -y + z + \mu(y) = 0 \tag{15}
$$

$$
\frac{\partial\mathcal{L}}{\partial\theta} = -\kappa + \int\mu(y)e(y)\frac{\partial f}{\partial\theta}g(y)dy = 0 \tag{16}
$$

$$
\frac{\partial\mathcal{L}}{\partial e(y)} = -\rho'(e(y))g(y) + \mu(y)f(E,\theta)g(y) + \int\mu(y')e(y')\frac{\partial f}{\partial E}\frac{\partial E}{\partial e(y)}g(y')dy' = 0. \tag{17}
$$

The last equation can be further simplified, since $\frac{\partial E}{\partial e(y)} = l(y) = g(y)$, and we obtain:

$$-\rho'(e(y)) + \underbrace{\mu(y)f(E,\theta)}_{\text{direct effect}} + \underbrace{\int \mu(y')e(y')\frac{\partial f}{\partial E}g(y')dy'}_{\text{across-skill crowding effect}} = 0. \tag{18}$$

The key differences with the segmented market case are that: now the vacancy posting condition (16) is unique and depends on an average of skills on the labor market, while the search effort condition (18) presents an across-skill crowding effect.

To simplify notation, let us define $\bar{y}$ a search effort weighted average of skills such that:

$$\bar{y} - z = \int (y-z)e(y)g(y)dy \Big/ \int e(y)g(y)dy.$$

Equations (16) and (18) can be rewritten as

$$\kappa = E\frac{\partial f}{\partial \theta}(\bar{y}-z) = M_v(EL,v)(\bar{y}-z) \tag{19}$$

and

$$\begin{aligned}
\rho'(e(y)) &= f(E,\theta)(y-z) + E\frac{\partial f}{\partial E}(\bar{y}-z) \\
&= \left(f(E,\theta) + E\frac{\partial f}{\partial E}\right)(y-z) - E\frac{\partial f}{\partial E}(y-\bar{y}) \\
&= M_{EL}(EL,v)(y-z) + \frac{\theta}{E}M_v(EL,v)(y-\bar{y})
\end{aligned} \tag{20}$$

where we have used the fact that $M(EL,v) = EL \cdot M_{EL} + vM_v$.

Let us now compare to the equilibrium conditions of the competitive economy:

$$\begin{aligned}
\kappa &= q(E,\theta)\int (1-\gamma)(y-z)e(y)g(y)dy \Big/ \int e(y)g(y)dy \\
\rho'(e(y)) &= f(E,\theta)\gamma(y-z).
\end{aligned}$$

We first notice that the two sets of conditions are in general different, meaning that the competitive equilibrium is not efficient in general. Under a standard Hosios condition $\gamma = EL \cdot \frac{M_{EL}(EL,v)}{M(EL,v)}$, these conditions become:

$$\kappa = M_v(EL,v)\int (y-z)e(y)g(y)dy \Big/ \int e(y)g(y)dy = M_v(EL,v)(\bar{y}-z) \tag{21}$$

$$\rho'(e(y)) = M_{EL}(EL,v)(y-z). \tag{22}$$

Notice that the Hosios condition already helps bring the competitive equilibrium closer to the constrained efficient allocation: the job-creation (or free-entry) condition (21) is the same as (19), the search effort condition (22) is the same as the first part of (20). This tells us that the Hosios condition is still useful to internalize the average congestion effect and succeeds in making firms post the efficient number of vacancies. The amount of search is, however, inefficient. We see from equation (20) that the constrained efficient allocation puts stronger incentives to search for skilled workers above $\bar{y}$ and reduces those of lower skills.

We are now going to prove that efficiency can be restored with a uniform tax on worker T, a linear wage subsidy s and a vacancy subsidy S_v to firms. Under these policies, the Nash bargaining procedure becomes:

$$w(y) = \underset{w}{\operatorname{argmax}}[w(1+s) - z - T]^{\gamma}[y - w]^{1-\gamma}. \tag{23}$$

Solving for the wage, we obtain

$$w(y) = \gamma y + \frac{1-\gamma}{1+s}(z+T),$$

and the corresponding search effort condition

$$\rho'(e(y)) = f(E,\theta)\Big(\gamma(1+s)y + (1-\gamma)(z+T) - z - T\Big).$$

Assuming that the Hosios condition $\gamma = EL \cdot \frac{M_{EL}(EL,v)}{M(EL,v)}$ holds, we must only choose s to align the search incentives. Choosing $s = 1/\gamma - 1$, we obtain the following search effort condition:

$$\rho'(e(y)) = f(E,\theta)\Big(y - z + (1-\gamma)z - \gamma T)\Big).$$

Going back to equation (20), we see that setting T so that

$$f(E,\theta)\Big((1-\gamma)z - \gamma T)\Big) \equiv E\frac{\partial f}{\partial E}(\bar{y} - z)$$

can implement the optimal search effort. Solving for T, we get:

$$T = -\frac{\frac{E\partial f}{\partial E}}{\gamma f}\bar{y} = \frac{\theta}{E}\frac{M_v}{M_{EL}}\bar{y} > 0.$$

Turning to the job creation condition (free-entry), we have:

$$\kappa = q(E,\theta)\frac{\int (y - w(y))e(y)g(y)dy}{\int e(y)g(y)dy} + S_v.$$

Plugging in the wage expression, we have:

$$\kappa = q(E,\theta)(1-\gamma)\frac{\int (y-z)e(y)g(y)dy}{\int e(y)g(y)dy} + (1-\gamma)(z(1-\gamma) - \gamma T) + S_v.$$

Choosing

$$S_v = -(1-\gamma)(z(1-\gamma) - \gamma T) = -(1-\gamma)\frac{\frac{E\partial f}{\partial E}}{f}(\bar{y} - z) = v^2\frac{M_v^2}{M^2}(\bar{y} - z) > 0.$$

is enough to restore an efficient level of job creation under the Hosios condition, as shown in (21).

□

Proof of prop.3:

(a) Assume $\{c_e(\cdot), T_f(\cdot), c_u\}$ is a truth-telling mechanism. In particular, $\forall y$:

$$y = \underset{\tilde{y}}{\operatorname{argmax}} [c_e(\tilde{y}) - c_u - z]^{\gamma}[y + T_f(\tilde{y})]^{1-\gamma}.$$

Implementation means finding a non-linear income tax τ and unemployment insurance b such that if we define

$$w(y) = \underset{w}{\operatorname{argmax}} [w - \tau(w) - b - z]^{\gamma}[y - w]^{1-\gamma},$$

then the same allocation is implemented, i.e :

$$\begin{cases} w(y) - \tau(w(y)) = c_e(y) \\ w(y) = -T_f(y) \\ b = c_u. \end{cases}$$

This implementation is feasible if

$$\forall y_1 \neq y_2, \quad T_f(y_1) = T_f(y_2) \Rightarrow c_e(y_1) = c_e(y_2).$$

Assume, by contradiction, that there exist $y_1 \neq y_2$ with $T_f(y_1) = T_f(y_2)$ but $c_e(y_1) < c_e(y_2)$. This trivially produces a contradiction, because agent 1 would always prefer to declare type y_2 instead of y_1:

$$[c_e(y_2) - c_u - z]^{\gamma}[y + T_f(y_2)]^{1-\gamma} > [c_e(y_1) - c_u - z]^{\gamma}[y + T_f(y_1)]^{1-\gamma}.$$

This violates our assumption about a truth-telling mechanism.

(b) Showing that $w(y)$ and $w(y) - \tau(w(y))$ are increasing functions of y is equivalent to show that $c_e(y)$ is increasing and $T_f(y)$ decreasing. Let $y_1 < y_2$. Since the mechanism is truth-telling, we know that either:

1. $c_e(y_1) = c_e(y_2)$ & $T_f(y_1) = T_f(y_2)$
2. $c_e(y_1) < c_e(y_2)$ & $T_f(y_1) > T_f(y_2)$
3. $c_e(y_1) > c_e(y_2)$ & $T_f(y_1) < T_f(y_2)$

Otherwise one type would always be preferred over the other one and the mechanism would not be truth-telling. We are going to show that the third inequality cannot satisfy the incentive constraint.

Assume by contradiction that $c_e(y_1) > c_e(y_2)$ & $T_f(y_1) < T_f(y_2)$. Define the following function

$$\Pi(y, c_e, T_f) \equiv [c_e - c_u - z]^{\gamma}[y + T_f]^{1-\gamma}.$$

Truthful-telling tells us that for agents of type y_2:

$$\Pi(y_2, c_e(y_2), T_f(y_2)] \geq \Pi(y_2, c_e(y_1), T_f(y_1)] > \Pi(y_1, c_e(y_1), T_f(y_1)]$$

The indifference curve for agent 1 in the (c_e, T_f) space is:

$$T_f = \Pi(y_1, c_e(y_1), T_f(y_1))^{1/1-\gamma}[c_e - c_u - z]^{-\gamma/1-\gamma}$$

The indifference of curve for agent 2 going through $(c_e(y_1), T_f(y_1))$ is

$$T_f = \Pi(y_2, c_e(y_1), T_f(y_1))^{1/1-\gamma}[c_e - c_u - z]^{-\gamma/1-\gamma}.$$

At the intersection of the two curves in $(c_e(y_1), T_f(y_1))$, we have:

$$\underbrace{\Pi(y_2, c_e(y_1), T_f(y_1))^{1/1-\gamma}}_{>\Pi(y_1, c_e(y_1), T_f(y_1))}[c_e - c_u - z]^{-\gamma/1-\gamma} = \Pi(y_1, c_e(y_1), T_f(y_1))^{1/1-\gamma}[c_e - c_u - z]^{-\gamma/1-\gamma}.$$

We have a form of single-crossing property: the indifference curves are decreasing with c_e, but that of agent 2 is steeper than for agent 1. Therefore, given that $c_e(y_2) < c_e(y_1)$ and that agent 2 chooses to declare type y_2 over type y_1:

$$\begin{aligned} T_f(y_2) &\geq \Pi(y_2, c_e(y_1), T_f(y_1))^{1/1-\gamma}[c_e(y_2) - c_u - z]^{-\gamma/1-\gamma} \\ &> \Pi(y_1, c_e(y_1), T_f(y_1))^{1/1-\gamma}[c_e(y_2) - c_u - z]^{-\gamma/1-\gamma}. \end{aligned}$$

We have a contradiction because agent 1 would like to deviate and claim to be of type 2. Figure 6 illustrates the argument and shows why c_e is increasing, while T_f is decreasing.

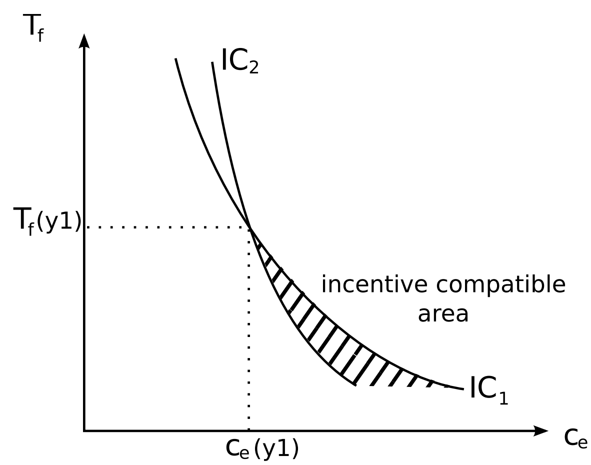

Figure 6: Single-crossing property

(c) In the eventual case where wages are negative in the optimal policy, we provide another implementation result with positive wages and a uniform hiring subsidy H to firms, on top of the non-linear income tax and unemployment insurance b. Denote $\tau(\cdot)$ and b the original

policy that produces negative wages and write $\tilde{H}$, $\tilde{\tau}(\cdot)$ and $\tilde{b}$ the new policy with positive wages.
Set:

$$
\begin{aligned}
\tilde{H} &= -\min_{y} w(y) \geq 0, \\
\tilde{\tau}(\tilde{w}) &= \tau(\tilde{w} - \tilde{H}) + \tilde{H}, \quad \forall \tilde{w}.
\end{aligned}
$$

Then the two allocations coincide for $\tilde{w}(y) = w(y) + \tilde{H}$:

$$
\begin{aligned}
&\tilde{w} - \tilde{\tau}(\tilde{w}) = \tilde{w} - \tau(\tilde{w} - \tilde{H}) - \tilde{H} = w - \tau(w), \\
&y - \tilde{w} + \tilde{H} = y - w.
\end{aligned}
$$

The two allocations are therefore identical and implement the allocation, but $\forall y, \tilde{w}(y) \geq 0$.
$\square$