Cascades and Fluctuations in an Economy with an Endogenous Production Network

Mathieu Taschereau-Dumouchel

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Cornell University

Introduction

- Firms rely on complex supply chains to get intermediate inputs
- These chains are constantly disrupted by suppliers going out of business
- Exit of one firm can push its suppliers and customers to exit
  - Cascade of firm failures
- These cascades change the structure of the production network
  - Affect how micro shocks aggregate into macro fluctuations

How do the entry/exit decisions of the firms affect the structure of the production network and aggregate fluctuations?

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Approach and results

Firms are connected with a finite set of suppliers/customers

- Fixed cost to operate → Firms operate or not depending on economic conditions
- Links between firms are active or not → Changes to the structure of the network

Key economic force: Complementarities in operation decisions of nearby firms

Efficient organization of production

- Tight clusters centered around productive firms
- A small change can trigger large reorganization of the network

Cascades of firm shutdowns

- Well-connected firms are hard to topple but create big cascades

Aggregate fluctuations

- Recessions feature fewer well-connected firms and less clustering

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Why study this problem

- Global survey of small and medium firms
  - 39% report that losing their main supplier would adversely affect their operation, and 14% would need to significantly downsize their business, require emergency support or shut down (Zurich Insurance Group, 2015)
- Fall 2008: carmakers are on the verge of bankruptcy
  - Policymakers worry about cascading effects through supply chains
  - Ford CEO calls for bailout of GM and Chrysler in Senate testimony
- Do entry/exit decisions matter for the shape of the network?
  - US data: 20% to 40% of link destructions occur with exit of supplier or customer

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Model

Model

- There are n firms that produce a differentiated good that can be used in the
  - production of a final good

$$C \equiv \left(\sum_{j=1}^{n} \beta_j^{\frac{1}{\sigma}} c_j^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

  - production of other differentiated goods
- Representative household
  - Consumes the final good
  - Supplies L units of labor inelastically

Model

- Firm j produces good j with the production function

$$y_j = \frac{A}{\alpha_j^{\alpha_j}(1-\alpha_j)^{1-\alpha_j}} z_j \theta_j \left(\sum_{i=1}^{n} \Omega_{ij}^{\frac{1}{\varepsilon_j}} x_{ij}^{\frac{\varepsilon_j - 1}{\varepsilon_j}} \right)^{\frac{\varepsilon_j}{\varepsilon_j - 1}\alpha_j} l_j^{1-\alpha_j}$$

- Firm j can only use good i as input if there is a connection from firm i to j
  - $\Omega_{ij} > 0$ if connection and $\Omega_{ij} = 0$ otherwise
  - A connection can be active or inactive
  - Matrix Ω is *exogenous*

- A firm can only produce if it pays a fixed cost $f_j L$ in units of labor
  - $\theta_j = 1$ if j is operating and $\theta_j = 0$ otherwise
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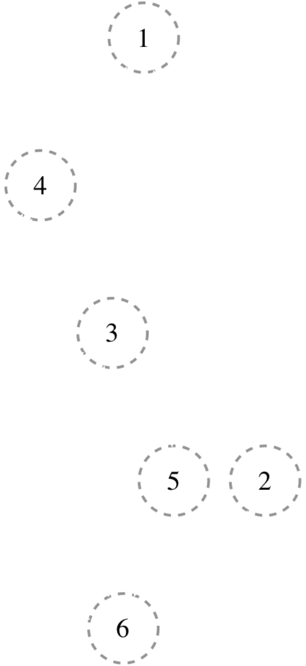
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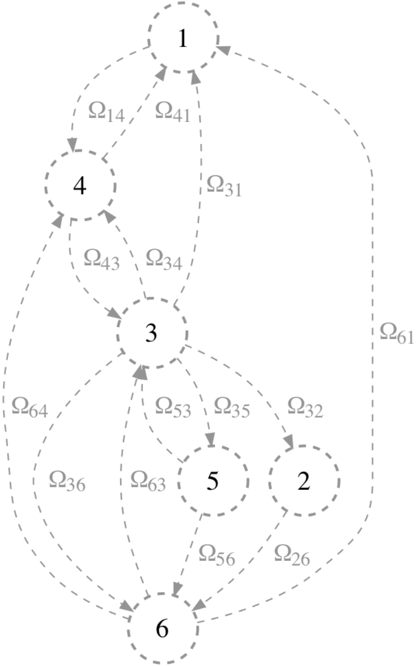
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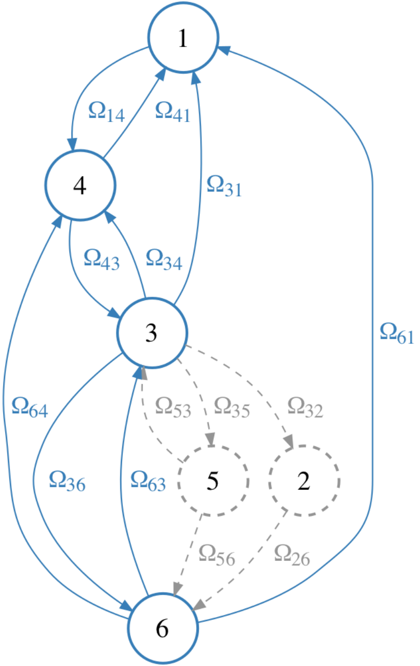
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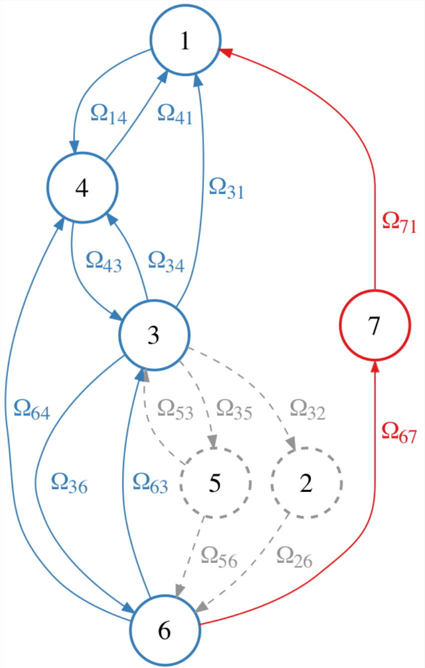
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Efficient allocation and equilibrium

For today: focus on the problem of a social planner

In the paper: different equilibrium definitions

1. Variations of monopolistic competition
2. Stable equilibria (Hatfield et al. 2013, Oberfield 2018).
   - An allocation is stable if there exist no coalition of firms that wishes to deviate.

Proposition

Every stable equilibrium is efficient.

▸ Stable equilibrium

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Social planner

Problem $\mathcal{P}$ of a social planner

$$\max_{\substack{c,x,l \\ \theta\in\{0,1\}^n}} \left(\sum_{j=1}^{n} \beta_j^{\frac{1}{\sigma}} c_j^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

subject to

1. a resource constraint for each good j

$$c_j + \sum_{k=1}^{n} x_{jk} \leq \frac{A}{\alpha_j^{\alpha_j}(1-\alpha_j)^{1-\alpha_j}} z_j \theta_j \left(\sum_{i=1}^{n} \Omega_{ij}^{\frac{1}{\varepsilon_j}} x_{ij}^{\frac{\varepsilon_j - 1}{\varepsilon_j}} \right)^{\alpha_j \frac{\varepsilon_j}{\varepsilon_j - 1}} l_j^{1-\alpha_j}$$

2. a resource constraint for labor

$$\sum_{j=1}^{n} l_j + \sum_{j=1}^{n} \theta_j f_j L \leq L$$

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1. a resource constraint for each good j **LM:** λ_j

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2. a resource constraint for labor **LM:** w

$$\sum_{j=1}^{n} l_j + \sum_{j=1}^{n} \theta_j f_j L \leq L$$

Social planner with exogenous θ

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Define $q_j = w/\lambda_j$

- From the FOCs, output is $(1 - \alpha_j)\, y_j = q_j l_j$
- q_j is the labor productivity of firm j

Proposition

In the efficient allocation

$$q_j = z_j \theta_j A \left(\sum_{i=1}^{n} \Omega_{ij} q_i^{\varepsilon_j - 1} \right)^{\frac{\alpha_j}{\varepsilon_j - 1}} \tag{1}$$

for all $j \in \mathcal{N}$. Furthermore, there is a unique vector q that satisfies (1).

$$q_j = z_j \theta_j A \left(\sum_{i=1}^{n} \Omega_{ij} q_i^{\varepsilon_j - 1} \right)^{\frac{\alpha_j}{\varepsilon_j - 1}}$$

- Access to a larger set of inputs increases productivity q_j
- Access to cheaper inputs (lower $1/q_i$) leads to a cheaper output
- Gains in productivity propagate downstream through supply chains

Key economic force: Gains from input variety

$$q_j = z_j \theta_j A \left(\sum_{i=1}^{n} \Omega_{ij} q_i^{\varepsilon_j - 1} \right)^{\frac{\alpha_j}{\varepsilon_j - 1}}$$

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Social planner with exogenous θ

With q we can solve for all other quantities easily

Lemma

Aggregate consumption is

$$C = Q\left(L - \sum_{j=1}^{n} \theta_j f_j L\right)$$

where $Q \equiv \left(\sum_{j=1}^{n} \beta_j q_j^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$ *is aggregate labor productivity.*

▸ Other quantities

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Planner's problem $\mathcal{P}$ can be expressed in terms of θ only

$$\max_{\theta\in\{0,1\}^n} Q\left(L-\sum_{j=1}^{n}\theta_j f_j L\right)$$

with

$$q_j = z_j\theta_j A\left(\sum_{i=1}^{n}\Omega_{ij}q_i^{\varepsilon_j-1}\right)^{\frac{\alpha_j}{\varepsilon_j-1}}$$

Trade-off: making firm j produce $(\theta_j = 1)$

- increases labor productivity of the network Q
- reduces the amount of labor into production $L-\sum_{j=1}^{n}\theta_j f_j L$

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"Hard" problem (MINLP — NP Hard)

1. Feasible set $\theta \in \{0,1\}^n$ is not convex
2. Objective function is not concave

Brute force approach: exhaustive search

- Take a $\theta \in \{0,1\}^n$, iterate on q and evaluate the objective function
- 2^n vectors θ to try ($\approx 10^6$ configurations for 20 firms)
- Guaranteed to find correct solution but infeasible for n large

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Alternative approach

New solution approach: Find an alternative problem such that

P1 The alternative problem is easy to solve

P2 A solution to the alternative problem also solves $\mathcal{P}$

Reshaping $\mathcal{P}$

Consider the relaxed and reshaped problem $\mathcal{R}$

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with

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Parameters $a_j > 0$ and b_{ij} reshape the objective function away from optimum (i.e. when $0 < \theta_j < 1$)

- For a_j: if $\theta_j \in \{0,1\}$ then $\theta_j^{a_j} = \theta_j$
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For $\theta \in \{0, 1\}^n$, a_j and b_{ij} do not affect the value of the planner's objective function

How to pick a_j and b_{ij}?

We are free to pick a_j and b_{ij} to help us solve $\mathcal{R}$

- Increase the concavity of $\mathcal{R}$ to remove local maxima
- But too much concavity might create new maximum in the middle of $[0, 1]^n$

Economic intuition: first-order condition of $\mathcal{R}$ with respect to θ_j

But thinking at the margin is misleading!

- We want the planner to compare the whole discrete change between $\theta = 0$ and $\theta = 1$

The parameters a_j and b_{ij} change the perceived value of good j when determining θ_j

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$$\lambda_j c_j + \sum_{k=1}^{n} \lambda_j x_{jk} - \sum_{i=1}^{n} \lambda_i x_{ij} - w l_j - w\theta_j f_j L = \theta_j \Delta\mu_j,$$

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But thinking at the margin is misleading!

- We want the planner to compare the whole discrete change between $\theta = 0$ and $\theta = 1$

The parameters a_j and b_{ij} change the perceived value of good j when determining θ_j

How to pick a_j and b_{ij}?

We are free to pick a_j and b_{ij} to help us solve $\mathcal{R}$

- Increase the concavity of $\mathcal{R}$ to remove local maxima
- But too much concavity might create new maximum in the middle of $[0,1]^n$

Economic intuition: first-order condition of $\mathcal{R}$ with respect to θ_j with reshaping

$$(1 + a_j)\,\lambda_j c_j + \sum_{k=1}^{n} (1 + a_j + b_{jk})\,\lambda_j x_{jk} - \sum_{i=1}^{n} \lambda_i x_{ij} - w l_j - w\theta_j f_j L = \theta_j \Delta\mu_j,$$

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What is the full gain in utility from operating j?

$$\Delta C = \int_0^{c_j} \frac{\partial C}{\partial c_j} d\tilde{c}_j = \int_0^{c_j} \beta_j^{\frac{1}{\sigma}} \tilde{c}_j^{-\frac{1}{\sigma}} C^{\frac{1}{\sigma}} d\tilde{c}_j = \frac{\sigma}{\sigma - 1} c_j \underbrace{\frac{\partial C}{\partial c_j}}_{\lambda_j}$$

The benefit of operating j should be proportional to $\frac{\sigma}{\sigma-1}$. Similar reasoning for b_{ij}.

From now on set

$$a_j = \frac{1}{\sigma - 1} \quad \text{and} \quad b_{ij} = \frac{1}{\varepsilon_j - 1} - \frac{1}{\sigma - 1} \qquad (\star)$$

and verify that these parameter values are helpful

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P1: Under some conditions the reshaped problem $\mathcal{R}$ is easy to solve

Proposition

Let $\varepsilon_j = \varepsilon$ and $\alpha_j = \alpha$ for all j. If $\Omega_{ij} = d_i e_j$ for some vectors d and e then the KKT conditions are necessary and sufficient to characterize a solution to $\mathcal{R}$.

Define $\bar{\Omega} = \omega(\mathbb{1} - I)$ where $\mathbb{1}$ is the all-one matrix, I the identity and $\omega > 0$.

Proposition

Let $\sigma = \varepsilon_j$ for all j. Suppose that the $\{\beta_j\}_{j\in\mathcal{N}}$ are not too far from each other and that the matrix Ω is close enough to $\bar{\Omega}$. Then there exists a threshold $\bar{f} > 0$ such that if $f_j < \bar{f}$ for all j the KKT conditions are necessary and sufficient to characterize a solution to $\mathcal{R}$.

These two propositions only provides sufficient conditions

- Later: robustness

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P2: A solution to $\mathcal{R}$ also solves $\mathcal{P}$

Proposition

If $\theta^* \in \{0,1\}^n$ solves $\mathcal{R}$, then θ^* also solves $\mathcal{P}$

But why would a solution to $\mathcal{R}$ be in $\{0,1\}^n$? First-order condition of $\mathcal{R}$ with respect to θ_j

- Under $(\star)$ the marginal benefit of θ_j only depends on θ_j through aggregates F_j and G_j
- For large connected network: $\{F_j, G_j\} \to$ independent of θ_j

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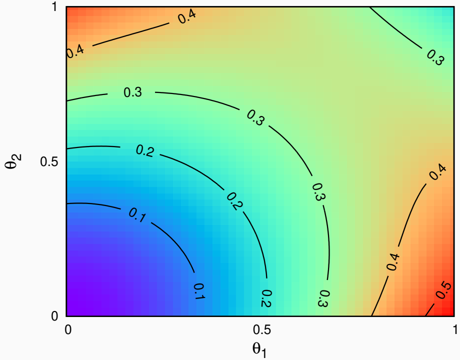
Example with two firms

Relaxed problem without reshaping



Problem: V is not concave

$\Rightarrow$ First-order conditions are not sufficient

$\Rightarrow$ Numerical algorithm can get stuck in local maxima

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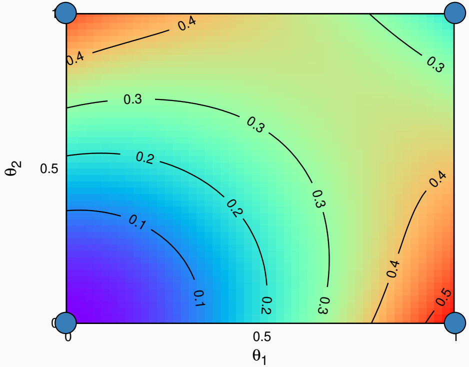

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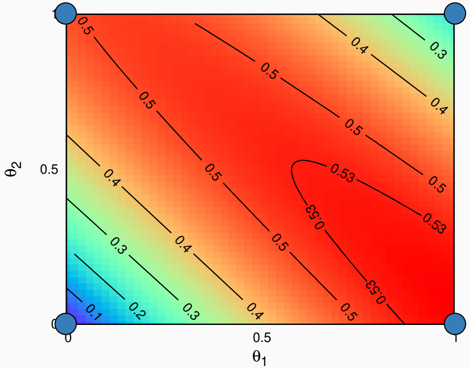


~~Problem~~: V is now (quasi) concave

$\Rightarrow$ First-order conditions are necessary and sufficient

$\Rightarrow$ Numerical algorithm converges to global maximum

Tests on Small Networks

For small networks we can solve $\mathcal{P}$ directly using exhaustive search and compare to solution of $\mathcal{R}$

	With reshaping		Without reshaping	
n	Correct θ	Error in C	Correct θ	Error in C
8	99.9%	0.001%	86.5%	0.791%
10	99.9%	0.001%	85.2%	0.855%
12	99.9%	0.001%	84.5%	0.903%
14	99.9%	0.001%	84.0%	0.926%

▸ Notes ▸ Break. by Ω ▸ Homo. firms ▸ Link by link ▸ Large networks
▸ Link by link large ▸ Error FOCs

The errors come from

1. firms that are particularly isolated
2. two θ configurations with almost same output

Tests with calibrated parameters

Same parameters as calibration

Table 1: Testing the reshaping approach for n large

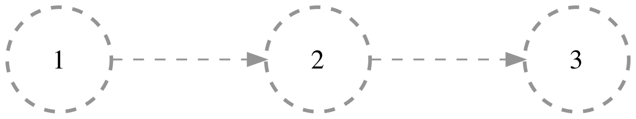
	With reshaping		Without reshaping	
n	Correct θ	Error in C	Correct θ	Error in C
1000	99.9%	$<$ 0.001%	66.5%	0.56%

Notes: Parameters as in the calibrated economy. We simulate 100 different matrices Ω and, for each Ω, draw 100 productivity vectors z. We run the procedure described in the appendix on each of them and report average results. $x < 0.001\%$ indicates that $x > 0$ but that proper rounding would yield 0.

Economic Forces

Gains from input variety create complementarities

Operating a firm increases the incentives to operate its neighbors in Ω.



- Impact of operating 2 on the incentives to operate 1 and 3
  - $\theta_2 = 1 \rightarrow q_2$ is larger if 1 operates
  - $\theta_2 = 1 \rightarrow q_3$ is larger if 3 operates
- Upstream and downstream complementarities in operating decisions
  - $\rightarrow$ Cascades of firm shutdowns
  - Stronger with low elasticity of substitution ε and higher input share α

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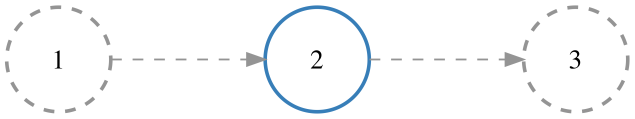

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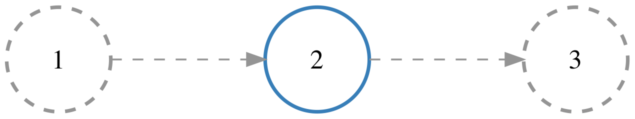

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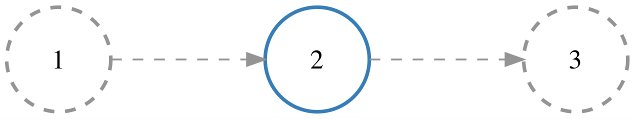

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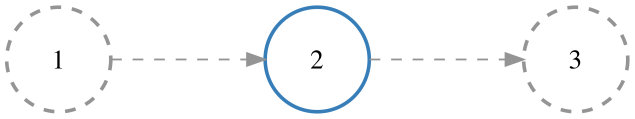

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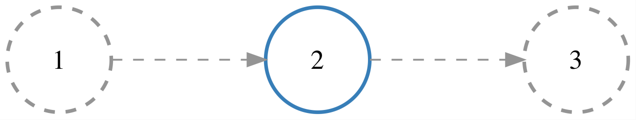

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Complementarities lead to clustering

Proposition

Operating a group of firms is more beneficial when there are more potential connections between them.

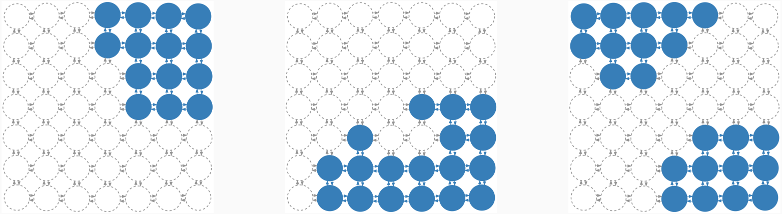
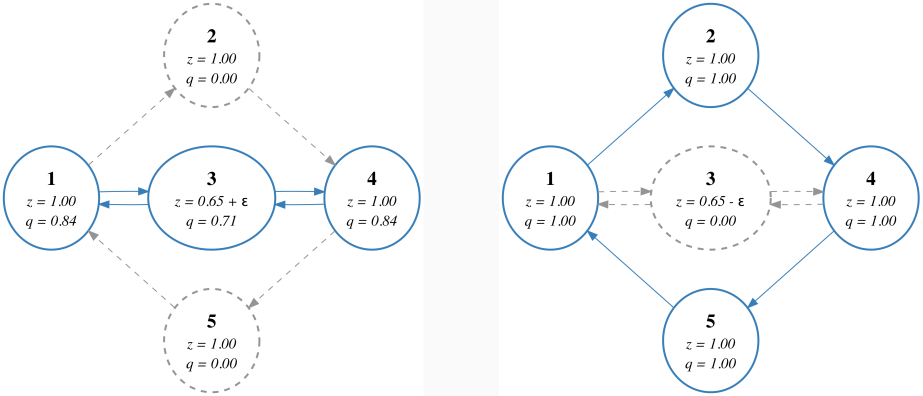


Figure 1: Clustering with three random draws of productivity z

▸ Formal statement

Large impact of small shock

Non-convex economy: a small shock can trigger a large reorganization



But welfare is barely affected (Theorem of the Maximum)

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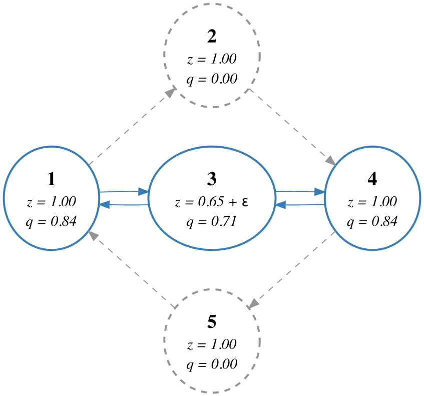

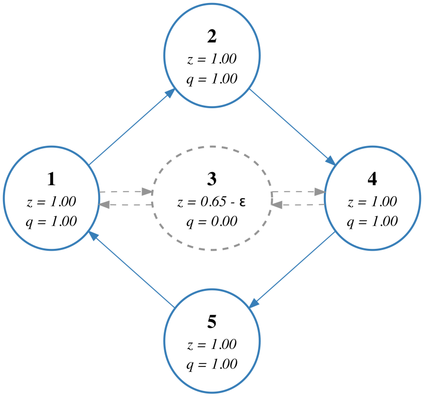

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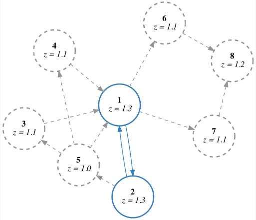
The role of elasticities



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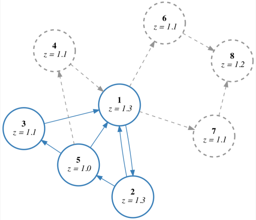

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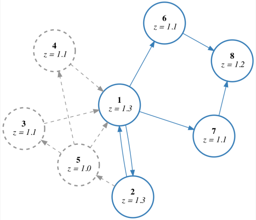

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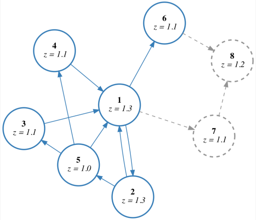

small ε, small σ

Quantitative Exploration

Network data

Two datasets that cover the U.S. economy

- Compustat
  - Public firms must self-report important customers ($>$10% of sales)
  - Cohen and al (2008) and Atalay et al (2011) use fuzzy-text matching algorithms to build the network
- Factset Revere
  - Includes public and private firms, and less important relationships
  - Data from 10-K, 10-Q, annual reports, investor presentations, websites, press releases, etc

	Years	Firms/year	Links/year
Compustat			
Atalay et al (2001)	1976 - 2009	1,300	1,500
Cohen and Frazzini (2006)	1980 - 2004	950	1,100
Factset	2003 - 2016	13,000	46,000

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Parameters

Focus on the shape of the network and limit heterogeneity across firms

Parameters from the literature

- $\alpha_j = 0.5$ to fit share of intermediate (Jorgenson et al 1987, Jones 2011)
- $\sigma = \varepsilon_j = 5$ average of estimates (Broda et al 2006)
- $\log z_{it}$ is AR1 with $\log z_{it} \sim$ iid $\mathcal{N}\left(0, 0.39^2\right)$ (Bartelsman et al, 2013), $\rho_z = 0.81$ (Foster et al, 2008)
- $f_j \times n = 5\%$ to fit employment in management occupations
- $n = 1000$ for high precision while limiting computations

Unobserved matrix Ω

- Picked to match the observed in-degree distribution
- Generate thousands of random Ω's and report averages

▸ Ω ▸ Cal. econ.

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Ω Cal. econ.

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Shape of the network

What does an optimally designed network looks like?

- Compare optimal and random networks
- Differences highlights how efficient allocation shapes the network

	Power law exponents		Clustering coefficient
Network	In-degree	Out-degree	
Efficient	0.97	0.92	3.45
Random	1.18	1.15	2.08

Efficient network has

- greater fraction of highly connected firms
- more clustering among firms

Def. clust. coeff.

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For each firm in each year

- Look at all neighbors upstream and downstream
- Regress the share of neighbors that exit on whether the original firm exits (and some controls)

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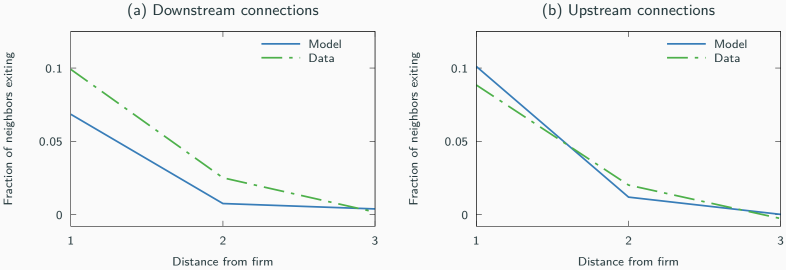

Resilience of firms

Size of cascades and probability of exit by degree of firm

	Size of cascades		Probability of exit	
	Data	Model	Data	Model
Average firm	0.9	1.1	11.8%	11.3%
High-degree firm	3.1	4.3	2.5%	1.7%

Notes: Size of cascades refers to firm exits up to and including the third neighbors. High degree means above the 90th percentile.

- Highly-connected firms are hard to topple but upon shutting down they create large cascades

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Aggregate fluctuations

Static model but z shocks move output and the structure of network together

Table 2: Correlations with aggregate output

	Model	Datasets		
		Factset	Compustat	
			AHRS	CF
Power law exponents				
In-degree distribution	−0.53	−0.87	−0.35	−0.12
Out-degree distribution	−0.63	−0.97	−0.31	−0.11
Global clustering coefficient	0.60	0.76	0.18	0.11

- Recessions: too costly to organize clusters around most productive firms

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Aggregate fluctuations

Static model but z shocks move output and the structure of network together

Table 2: Correlations with aggregate output

	Model	Datasets		
		Factset	Compustat	
			AHRS	CF
Power law exponents				
In-degree distribution	−0.53	−0.87	−0.35	−0.12
Out-degree distribution	−0.63	−0.97	−0.31	−0.11
Global clustering coefficient	0.60	0.76	0.18	0.11

- Recessions: too costly to organize clusters around most productive firms

Aggregate fluctuations

$$Y = Q\left(L - \sum_j \theta_j f_j L\right)$$

Table 3: Standard deviations of log aggregates

	Output		Labor Prod.		Prod. labor
	Y	$\approx$	Q	$+$	$L - \sum_j f_j \theta_j$
Optimal network	0.10		0.10		0.009
Fixed network	0.12		0.12		0

- Volatility of output about 20% smaller when network evolves endogenously
  - The difference comes from changes in the structure of the network
- Average output is also 11% lower

Intuition

Aggregate fluctuations

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▸ Intuition

Conclusion

Summary

- Model of network formation through entry/exit of firms
- Complementarities lead to clustering of activity and cascades
- Calibration captures empirical cascades and correlation between network and output
- Reorganization of network leads to smaller fluctuation

In the paper: inefficient allocations

- Reshaping can also solve those equilibrium
- Different upstream/downstream complementarities
- More rigid networks

Appendix

Stable equilibrium

- Definitions
  - A *contract* between i and j is a quantity shipped x_{ij} and a payment T_{ij}.
  - An *arrangement* is a contract between all possible pairs of firms.
  - A *coalition* is a set of firms J.
  - A *deviation* for a coalition J consists of
    1. dropping any contracts with firms not in J and,
    2. altering any contract involving two firms in J.
  - A *dominating deviation* is a deviation such that no firm is worse off and one firm is better off.
  - An allocation is *feasible* if $c_j + \sum_k x_{jk} \leq y_j$ and $\sum_j l_j + \theta_j f_j L \leq L$.

Stable equilibrium

- Firm j maximize profits

$$\pi_j = p_j c_j - w l_j + \sum_{i=1}^{n} T_{ji} - \sum_{i=1}^{n} T_{ij} - \theta_j w f_j L,$$

subject to $c_j + \sum_{k=1}^{n} x_{jk} \leq y_j$ and $c_j = \beta_j C(p_j/P)^{-\sigma}$.

Definition 1

A stable equilibrium is an arrangement $\{x_{ij}, T_{ij}\}_{i,j \in \mathcal{N}^2}$, firms' choices $\{p_j, c_j, l_j, \theta_j\}_{j \in \mathcal{N}}$ and a wage w such that:

1. the household maximizes,
2. firms maximize,
3. markets clear,
4. there are no dominating deviations by any coalition, and
5. the equilibrium allocation is feasible.

Return

Stable equilibrium

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Return

Other quantities

- Labor allocation

$$l = \left[(I_n - \Gamma)\,\mathrm{diag}\left(\frac{1}{1-\alpha}\right)\right]^{-1} \left(\beta \circ \left(\frac{q}{Q}\right)^{\circ(\sigma-1)} \frac{Y}{Q}\right)$$

- Output

$$(1-\alpha_j)\, y_j = q_j l_j$$

- Consumption

$$c_j = \beta_j \left(\frac{q_j}{w}\right)^{\sigma} Y$$

- Intermediate goods flows

$$x_{ij}\lambda_i^{\varepsilon_j} = \lambda_j^{\varepsilon_j}\alpha_j \left(A z_j \theta_j \left(\frac{\lambda_j}{w}\right)^{1-\alpha_j}\right)^{\frac{\varepsilon_j - 1}{\alpha_j}} \delta_{ij}\Omega_{ij}^{\varepsilon_j} y_j.$$

Return

Tests Details

Aggregates parameters

- $\sigma \in \{4, 6, 8\}$
- $\log(z_k) \sim$ iid $\mathcal{N}\left(0, 0.25^2\right)$
- Ω randomly drawn such that firms have on average 3, 4, 5, 6, 7 or 8 *potential* incoming connections
  - The corresponding average number of *active* incoming connections is 2.1, 3.0, 3.8, 4.5, 5.3, and 5.8, respectively.
  - For each non-zero: $\Omega_{ij} \sim$ iid $U([0, 1])$

Individual parameters

- $f_j \sim$ iid $U([0, 0.2/n])$
- $\alpha_j \sim$ iid $U([0.25, 0.75])$
- $\varepsilon_j \sim$ iid $U([4, \sigma])$
- $\beta_j \sim$ iid $U([0, 1])$

For each possible combination of aggregate parameters, 200 networks Ω and productivity vectors z are drawn. An economy is kept in the sample only if the first-order conditions yield a solution for which θ

Breakdown by Ω

		Firms with correct θ		
n	Reshaping?	All Ω's	More connected Ω's	Less connected Ω's
8	Yes	99.8%	99.9%	99.6%
	No	88.2%	89.1%	87.4%
10	Yes	99.7%	99.9%	99.5%
	No	86.5%	87.3%	85.8%
12	Yes	99.7%	99.9%	99.5%
	No	86.2%	87.0%	85.5%
14	Yes	99.7%	99.9%	99.4%
	No	85.5%	86.1%	85.1%

- Less connected Ω: firms have 3, 4 or 5 potential incoming connections
- More connected Ω: firms have 6, 7 or 8 potential incoming connections

Return

Homogeneous Firms

	Number of firms n			
	8	10	12	14
A. With reshaping				
Firms with correct θ	99.9%	99.8%	99.8%	99.8%
Error in output Y	0.001%	0.002%	0.002%	0.002%
B. Without reshaping				
Firms with correct θ	87.2%	85.8%	84.7%	83.8%
Error in output Y	0.71%	0.79%	0.85%	0.89%

Notes: Random networks with parameters $f \in \{0.05/n, 0.1/n, 0.15/n\}$, $\sigma_z = 0.25$, $\alpha \in \{0.45, 0.5, 0.55\}$, $\sigma \in \{4, 6, 8\}$, $\varepsilon \in \{4, 6, 8\}$ and networks Ω randomly drawn such that firms have on average 2, 4, 5, 6, 7 to 8 *potential* incoming connections. Each non-zero Ω_{ij} is set to 1. For each combination of the parameters, 200 different economies are created. For each economy, productivity is drawn from $\log(z_k) \sim$ iid $\mathcal{N}\left(0, \sigma_z^2\right)$. An economy is kept in the sample only if the first-order conditions yield a solution for which θ hits the bounds. More than 90% of the economies are kept in the sample.

Return

Link by link

- Real firms: $f_j = 0$, $\alpha_j = 0.5$, $\sigma = \varepsilon_j = 6$ and $\sigma_z = 0.25$
- Link firms: $\beta_j = 0$, only one input and one output, $f_j \sim$ iid $U([0, 0.1/n])$, $\alpha_j \sim$ iid $U([0.5, 1])$, $\sigma_z = 0.25$
- Ω: between any two real firm, there is a link firm with probability $p \in \{0.7, 0.8, 0.9\}$

Number of firms		With reshaping		Without reshaping	
Real firms m	Link firms $n - m$	Correct θ	Error in C	Correct θ	Error in C
3	up to 6	99.9%	0.001%	94.1%	0.17%
4	up to 12	99.7%	0.003%	91.3%	0.25%
5	up to 20	99.7%	0.006%	89.2%	0.31%

Return

Large Networks

For large networks we cannot solve $\mathcal{P}_{SP}$ directly by trying all possible vectors θ

- After all the welfare-improving 1-deviations θ are exhausted:

	With reshaping		Without reshaping	
n	Correct θ	Error in C	Correct θ	Error in C
1000	$> 99.9\%$	$< 0.001\%$	68.9%	0.58%

Notes: 200 different Ω and z that satisfy the properties of the calibrated economy.

- No guarantee that the solution has been found but very few “obvious errors”

Return

Link by link

- Same parameters as before
- After all the welfare-improving 1-deviation in θ are exhausted:

Number of firms		With reshaping		Without reshaping	
Real firms m	Link firms $n-m$	Correct θ	Error in C	Correct θ	Error in C
10	up to 90	99.7%	0.005%	83.8%	0.46%
25	up to 600	99.9%	0.001%	80.5%	0.55%
40	up to 1560	$<$ 99.9%	$<$ 0.001%	79.5%	0.57%

- θ_j converges on $\{0,1\}$ for all j in about 60-85% of the tests
  - Even without convergence small error in output and few errors in θ

Return

Solution away from corners

- Sometimes the first-order conditions do not converge on a corner.
- Without excluding these simulations:

		Error in C		
n	Reshaping?	All Ω's	More connected Ω's	Less connected Ω's
8	Yes	0.007%	$<$ 0.001%	0.014%
	No	0.683%	0.640%	0.726%
10	Yes	0.013%	$<$ 0.001%	0.027%
	No	0.781%	0.739%	0.823%
12	Yes	0.008%	$<$ 0.001%	0.016%
	No	0.799%	0.744%	0.853%
14	Yes	0.008%	0.001%	0.016%
	No	0.831%	0.801%	0.862%

Return

Formal statement

Proposition 1

Let $\mathcal{J} \subset \mathcal{N}$ be a group of firms. Denote by $\theta^+ \in \{0,1\}^n$ the operating vector when the firms in $\mathcal{J}$ operate ($\theta_j^+ = 1$ for $j \in \mathcal{J}$). Similarly, let $\theta^- \in \{0,1\}^n$ be the operating vector when the firms in $\mathcal{J}$ do not operate ($\theta_j^- = 0$ for $j \in \mathcal{J}$). For all $j \notin \mathcal{J}$, assume $\theta_j^+ = \theta_j^-$. Denote by Ω^- a network of potential connections and let Ω^+ be identical to Ω^- except that it has an additional connection between two firms in $\mathcal{J}$. Then

$$C_{\Omega^+}\left(\theta^+\right) - C_{\Omega^+}\left(\theta^-\right) \geq C_{\Omega^-}\left(\theta^+\right) - C_{\Omega^-}\left(\theta^-\right),$$

where $C_\Omega\left(\theta\right)$ denotes consumption under the potential network Ω and the operating vector θ.

Return

Clustering coefficient

- Ω is drawn randomly so that joint distribution of in-degree and out-degree is a bivariate power law of the first kind

$$f(x_{in}, x_{out}) = \xi\,(\xi - 1)\,(x_{in} + x_{out} - 1)^{-(\xi+1)}$$

  where ξ is calibrated to 1.85. The marginals for x_{in} and x_{out} follow power law with exponent ξ.

- Correlation between observed in-degree and out-degree
  - Model: 0.67
  - Data: 0.43

Return

Calibrated Network

	Model	Datasets		
		Factset	Compustat	
			AHRS	CF
Power law exponents				
In-degree distribution	0.97	0.97	1.13	1.32
Out-degree distribution	0.92	0.83	2.24	2.22
Global clustering coefficient (normalized)	3.45	3.46	0.08	0.09

Notes: Global clustering coefficients are multiplied by the square roots of the number of nodes for better comparison.

Return

Shape of Network

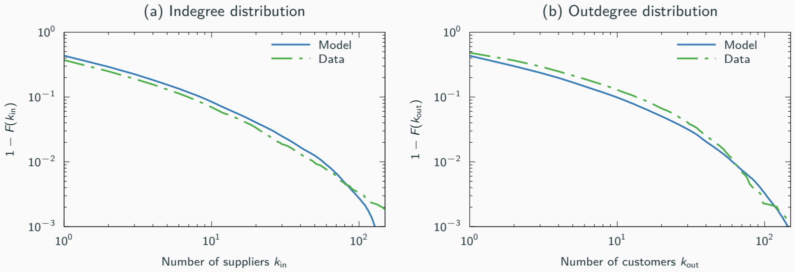


Figure 2: Model and Factset data for 2016

Return

Clustering coefficient

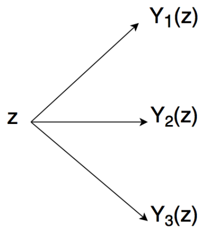
- Triplet: three connected nodes (might be overlapping)
- Triangles: three fully connected nodes (3 triplets)

$$\text{Clustering coefficient} = \frac{3 \times \text{number of triangles}}{\text{number of triplets}}$$

Return

Intuition

A given network θ^k is a function that maps $z \rightarrow Y_k(z)$



From extreme value theory:

$$\mathrm{Var}(Y) = \mathrm{Var}\left(\max_{k \in \{1,\dots,2^n\}} Y_k\right)$$

declines rapidly with n

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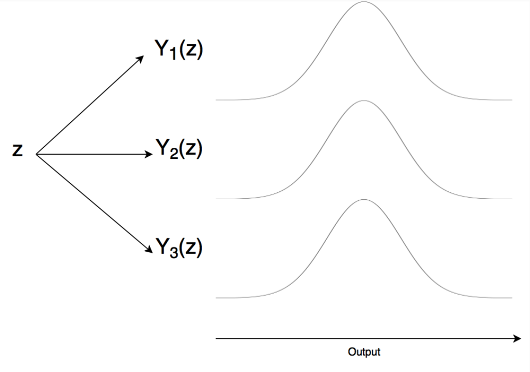

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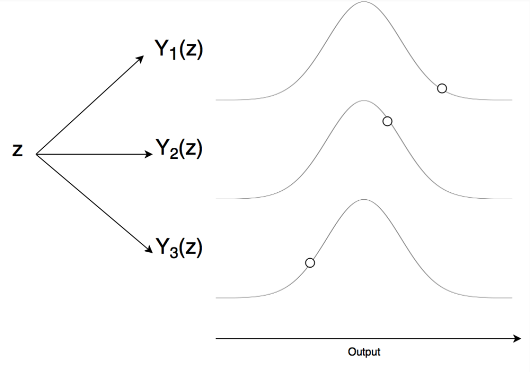

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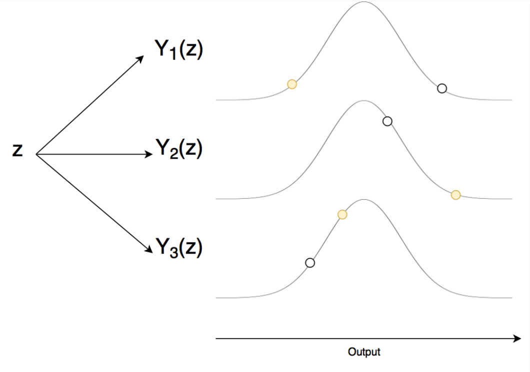

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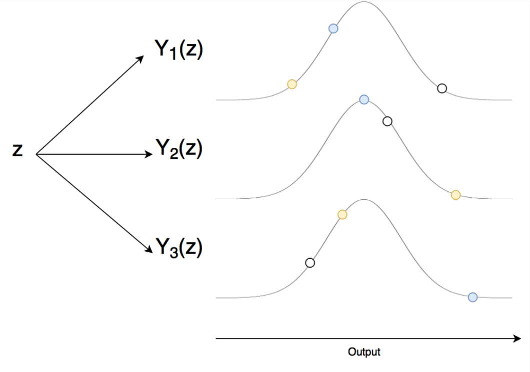

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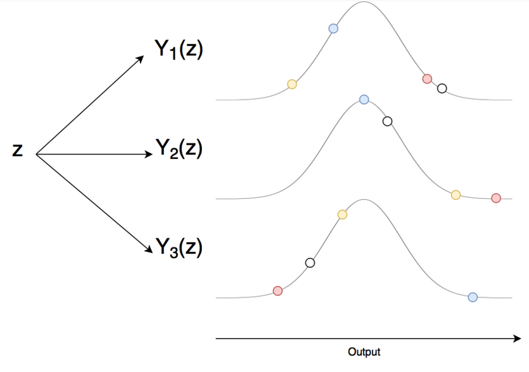

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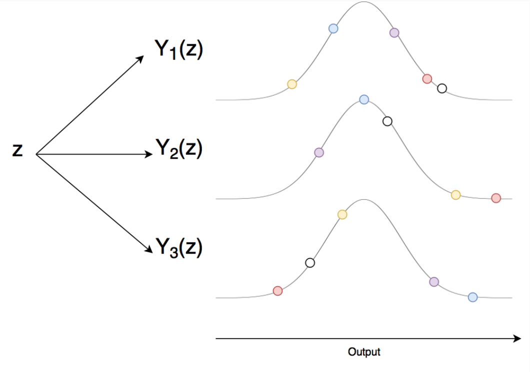

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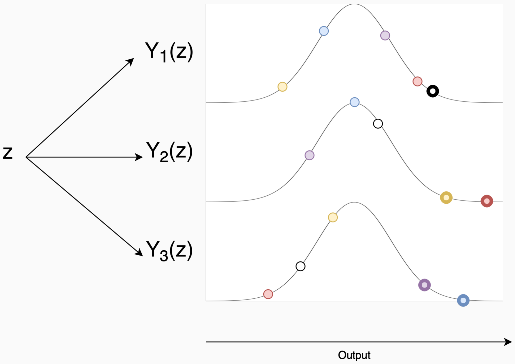

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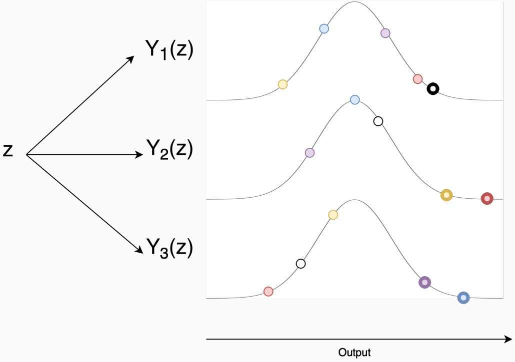

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