Echoes and Delays: Time-to-Build in Production Networks

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Motivation

- The Covid pandemic has shown how supply-chain disruptions and delays could shake up the world economy
- A large part of dynamic propagation of shocks through delays and time-to-build is ignored by production network literature
    - Acemoglu et al. (2012), Baqaee and Farhi (2019, 2020),... essentially static
    - Roundabout production: disruptions are resolved within period
- How does the introduction of time-to-build or delivery lags affect dynamics of production networks?

What We Do

- We propose a simple model to introduce time-to-build (T2B) in production networks
    - Long and Plosser (1983) (one period delay) + heterogeneous T2B
- We analyze how T2B contributes to propagation of shocks:
    1. Persistence, delays and bottlenecks
    2. Echoes and endogenous fluctuations
    3. Dynamic sectoral comovements and aggregation
- Empirical evidence (in progress)

Data

Input-Output

- IO-Use tables from BEA for 2017
    - 402 6-digit NAICS industries

Time-to-Build

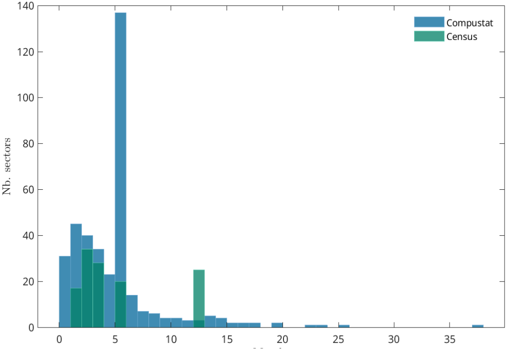
- Measure:

$$\text{backlog ratio} = \frac{\text{stock value of unfilled orders}}{\text{flow value of goods delivered}}$$

- US Census M3 survey on "Shipments, Inventories and Orders" (monthly)
    - All manufacturing, aggregated to $\sim$ 10 subsectors for 1992-2024
- Compustat "Order Backlog" variable (annual)
    - Publicly listed firms but firm level & broader sectoral coverage for 1970-2024

▸ Intuition

Backlog Distribution



Note: unweighted, mean $=4.74$ months, average $\sigma\left(\widehat{\text{backlog}_t}\right) = 0.5530$

Figure 1: Distribution of backlog ratios (months) across 6-digit sectors

Model

Model

- Time is discrete
- Representative household with inelastic labor supply
- Sectors $i = 1, \ldots, N$ with production

$$y_{it} = A_{it} F_i \left(l_{it}, x_{i1,t}, ..., x_{iN,t} \right)$$

- Time-to-build modeled as delivery lags:
  - Goods in sector i take τ_i periods to be delivered
  - Denote $X_{i\tau} \equiv$ agg. stock of i scheduled for delivery in τ periods
- Ignore inventories for now

Planning Problem

- **Planning problem**:

$$V\left(\left\{A_i\right\}, \left\{X_{1\tau}\right\}_{t=0}^{\tau_1-1}, \ldots, \left\{X_{N\tau}\right\}_{\tau=0}^{\tau_N-1}\right) = \max_{c_i, l_i, x_{ij}, y_i} U\left(c_1, \ldots, c_N\right) + \beta E\left[V\left(\left\{A_i'\right\}, \left\{X_{1\tau}'\right\}_{t=0}^{\tau_1-1}, \ldots, \left\{X_{N\tau}'\right\}_{\tau=0}^{\tau_N-1}\right)\right]$$

subject to:

$$1 \geq \sum_{i=1}^{N} l_i$$

and for all $i = 1..N$:

$$X_{i\tau}' = X_{i\tau+1} \text{ for } 0 \leq \tau < \tau_i - 1$$

$$X_{i\tau_i-1}' = y_i$$

$$X_{i0} \geq c_i + \sum_j x_{ji}$$

$$y_i = A_i F_i\left(l_{it}, x_{i1}, ..., x_{iN}\right)$$

Planning Problem

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X_{i\tau_i-1}' &= y_i\\
X_{i0} &\geq c_i+\sum_j x_{ji}\\
y_i &= A_iF_i\left(l_{it},x_{i1},\ldots,x_{iN}\right)
\end{aligned}$$

Planning Problem

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$$X_{i\tau_i-1}'=y_i$$
$$X_{i0}\geq c_i+\sum_j x_{ji}$$
$$y_i=A_iF_i\left(l_{it},x_{i1},\ldots,x_{iN}\right)$$

Solution

- High dimensional state space: 402 sectors × # lags !
- But a special case has an analytical solution:

Proposition
For $F_i(l, x_1, \ldots, x_N) = l^{\alpha_i} \prod_{j=1}^{N} x_{ij}^{\omega_{ij}}$ *for* $\alpha_i + \sum_j \omega_{ij} = 1$ *and* $U(c_1, \ldots, c_N) = \sum_1^N \gamma_i \log c_i$, *the economy can be solved analytically*

$$V(\mathbf{A}, \mathbf{X}_1, \ldots) = \sum_{i=1}^{N} \sum_{\tau=0}^{\tau_i} \beta^{\tau} \zeta_i \log X_{i\tau} + G(\mathbf{A})$$

where

$$\boldsymbol{\zeta} = \left(I - \left[\boldsymbol{\Omega} \cdot \beta^{\boldsymbol{\tau}}\right]'\right)^{-1} \boldsymbol{\gamma}$$
$$G(\mathbf{A}) = \sum_i \beta^{\tau_i} \zeta_i \log A_i + \beta E\left[G(\mathbf{A}')\right]$$

and the allocation satisfies

$$c_i = \overline{c_i} X_{i0}$$
$$x_{ij} = \overline{x_{ij}} X_{io}$$
$$l_i = \overline{l_i}$$

Decentralization

- Decentralization:
  - Spot price (immediate delivery):

$$p_{it} \equiv \frac{\zeta_i}{X_{i0}\left(t\right)}$$

  - Futures: price at t for delivery at $t + \tau$

$$p_{it+\tau|t} \equiv \beta^{\tau} \frac{\zeta_i}{X_{i\tau}\left(t\right)}$$

Domar Weights and Hulten Theorem

- ζ corresponds to the Domar weights: for $VA_t = \sum p_{it} c_{it}$,

$$\begin{aligned}\zeta_i &= \frac{p_{it} X_{i0}(t)}{VA_t} = \frac{p_{it} y_{it-\tau_i}}{VA_t} = \frac{p_{it}\left(c_{it} + \sum_j x_{ji,t}\right)}{VA_t} \\ &= \gamma_i + \sum_j \omega_{ji} \beta^{\tau_j} \zeta_j\end{aligned}$$

$$\Rightarrow \boxed{\boldsymbol{\zeta} = \left(I - [\boldsymbol{\Omega} \cdot \beta^{\boldsymbol{\tau}}]'\right)^{-1} \boldsymbol{\gamma}}$$

- A horizon-adjusted version of Hulten theorem applies:

$$\frac{\partial V}{\partial \log A_i} = \beta^{\tau_i} \zeta_i$$

  - V is welfare, not real GDP
  - β^{τ_i} is time adjustment for delayed delivery

Domar Weights

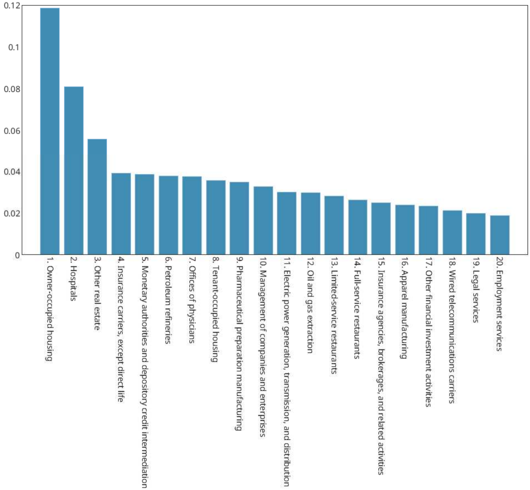


Figure 2: Top-20 sectors by Domar weight (Compustat)

Output

- In log-deviation from steady state:

$$\boxed{\hat{y}_{it} = \hat{A}_{it} + \sum_j \omega_{ij} \hat{y}_{jt-\tau_j}}$$

- VAR(τ_{max}) representation:

$$\hat{\mathbf{y}}_t = \mathbf{\Omega}_1 \hat{\mathbf{y}}_{t-1} + \ldots + \mathbf{\Omega}_{\tau_{max}} \hat{\mathbf{y}}_{t-\tau_{max}} + \hat{\mathbf{A}}_t$$

  where $\mathbf{\Omega}_\tau = \mathbf{\Omega} \cdot \mathbf{1}\left\{\tau = \tau_i\right\}$

- Nested cases:
  - Roundabout production:

$$\begin{aligned} \hat{\mathbf{y}}_t = \hat{\mathbf{A}}_t + \mathbf{\Omega}\hat{\mathbf{y}}_t \Rightarrow \hat{\mathbf{y}}_t &= \left[\mathbf{I} - \mathbf{\Omega}\right]^{-1} \hat{\mathbf{A}}_t \quad \text{(Leontieff inverse)} \\ &= \hat{\mathbf{A}}_t + \mathbf{\Omega}\hat{\mathbf{A}}_t + \mathbf{\Omega}^2\hat{\mathbf{A}}_t + \ldots \end{aligned}$$

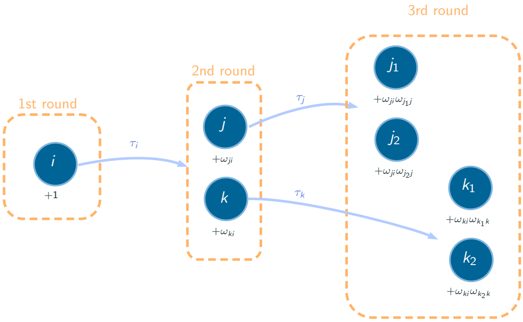
  - Long and Plosser (1983):

$$\hat{\mathbf{y}}_t = \hat{\mathbf{A}}_t + \mathbf{\Omega}\hat{\mathbf{y}}_{t-1} \Rightarrow \hat{\mathbf{y}}_t = \hat{\mathbf{A}}_t + \mathbf{\Omega}\hat{\mathbf{A}}_{t-1} + \mathbf{\Omega}^2\hat{\mathbf{A}}_{t-2} + \ldots$$

Persistence and Delay Shocks

Persistence Statistics

- Consider a shock to i at time t:



- Define the average duration of a shock:

$$\mathcal{T}\left(\hat{\mathbf{A}}\right) = \sum_{\tau=0}^{\infty} \sum_{i} \tau w_i \hat{y}_{i\tau}\left(\hat{\mathbf{A}}\right)$$

where w_i some weighting vector and $\hat{y}_{i\tau}\left(\hat{\mathbf{A}}\right)$ the IRF to shock $\hat{\mathbf{A}}$

Average Duration

Proposition

The average duration $\mathcal{T}\left(\hat{\mathbf{A}}\right)$ *for weighting vector* $\mathbf{w}$ *is equal to*

$$\mathcal{T}\left(\hat{\mathbf{A}}\right) = \mathbf{w}'\boldsymbol{\Omega}\left[\mathbf{I} - \boldsymbol{\Omega}\right]^{-1} \mathit{diag}\left(\boldsymbol{\tau}\right)\left[\mathbf{I} - \boldsymbol{\Omega}\right]^{-1}\hat{\mathbf{A}}$$

Intuition: Consider single shock $\boldsymbol{\delta}_i = \begin{pmatrix} 0 & \dots & 1 & \dots & 0 \end{pmatrix}'$ to sector i:

$$\mathcal{T}\left(\boldsymbol{\delta}_i\right) = \mathbf{w}' \underbrace{\boldsymbol{\Omega}}_{\substack{\text{duration } \tau \text{ only} \\ \text{contributes after 1 round}}} \underbrace{\left[\sum_{k=0}^{\infty}\boldsymbol{\Omega}^k\right]}_{\substack{\text{contribution of } \tau_j \text{ to} \\ \text{later rounds of production}}} \text{diag}\left(\boldsymbol{\tau}\right) \underbrace{\left[\sum_{k=0}^{\infty}\boldsymbol{\Omega}^k\right]\boldsymbol{\delta}_{\mathbf{i}}}_{\substack{\#\text{ of walks from sector } i \\ \text{to other sector} j \text{ of any length}}}$$

The rest follows by linearity to any shock $\hat{\mathbf{A}}$.

Average Duration

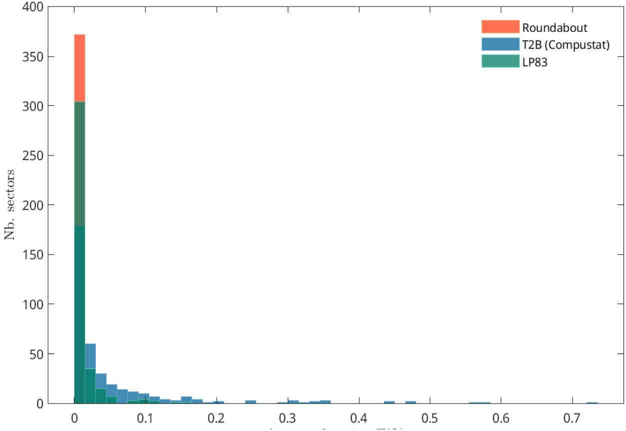


Figure 3: Comparison of average durations of *iid* sectoral shocks

Bottlenecks

Which sector's T2B contributes the most to propagation?

- Marginal impact of a delay shock on aggregate shock:

$$\frac{\partial \mathcal{T}(\mathbf{1})}{\partial \tau_i} = \mathbf{w}'\boldsymbol{\Omega}\left[\mathbf{I}-\boldsymbol{\Omega}\right]^{-1}\frac{\partial \mathrm{diag}(\boldsymbol{\tau})}{\partial \tau_i}\left[\mathbf{I}-\boldsymbol{\Omega}\right]^{-1}\mathbf{1}$$

Bottlenecks

Which sector's T2B contributes the most to propagation?

- Marginal impact of a delay shock on aggregate shock:

$$\frac{\partial \mathcal{T}\left(\mathbf{1}\right)}{\partial \tau_i} = \mathbf{w}'\mathbf{\Omega}\left[\mathbf{I}-\mathbf{\Omega}\right]^{-1}\begin{pmatrix} 0 & & & & \\ & \ddots & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 0 \end{pmatrix}\left[\mathbf{I}-\mathbf{\Omega}\right]^{-1}\mathbf{1}$$

Bottlenecks

Which sector's T2B contributes the most to propagation?

- Marginal impact of a delay shock on aggregate shock:

$$\frac{\partial \mathcal{T}(\mathbf{1})}{\partial \tau_i} = \mathbf{w}'\mathbf{\Omega}\left[\mathbf{I}-\mathbf{\Omega}\right]^{-1}\boldsymbol{\delta}_i\boldsymbol{\delta}_i'\left[\mathbf{I}-\mathbf{\Omega}\right]^{-1}\mathbf{1}$$

Bottlenecks

Which sector's T2B contributes the most to propagation?

- Marginal impact of a delay shock on aggregate shock:

$$\frac{\partial \mathcal{T}(\mathbf{1})}{\partial \tau_i} = \mathbf{w}'\boldsymbol{\Omega}\left[(\sum_{k=0}^{\infty}\boldsymbol{\Omega}^k)\boldsymbol{\delta}_i\right] \times \left[(\sum_{k=0}^{\infty}(\boldsymbol{\Omega}')^k)\boldsymbol{\delta}_i\right]'\mathbf{1}$$

Bottlenecks

Which sector's T2B contributes the most to propagation?

- Marginal impact of a delay shock on aggregate shock:

$$\frac{\partial \mathcal{T}(\mathbf{1})}{\partial \tau_i} = \mathbf{w}'\boldsymbol{\Omega}\left[\left(\sum_{k=0}^{\infty}\boldsymbol{\Omega}^k\right)\boldsymbol{\delta}_i\right] \times \left[\left(\sum_{k=0}^{\infty}(\boldsymbol{\Omega}')^k\right)\boldsymbol{\delta}_i\right]' \mathbf{1}$$

- Denote $\tilde{\mathbf{w}} = \boldsymbol{\Omega}'\mathbf{w}$:

$$\frac{\partial \mathcal{T}(\mathbf{1})}{\partial \tau_i} = \underbrace{\sum_j \tilde{w}_j \left[\sum_{k=0}^{\infty}\boldsymbol{\Omega}^k\right]_{ji}}_{\substack{\#\text{ of walks from } i \text{ to all sectors} \\ \text{of any length (weighted)}}} \times \underbrace{\sum_j \left[\sum_{k=0}^{\infty}(\boldsymbol{\Omega}')^k\right]_{ij}}_{\substack{\#\text{ of walks from all sectors } j \\ \text{to } i \text{ of any length}}}$$

⇒ Bottleneck in propagation = Supplier centrality × Buyer centrality

Bottlenecks

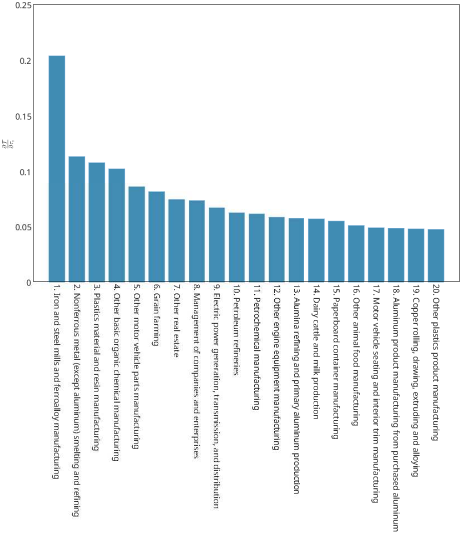


Figure 4: Top-20 bottleneck sectors

Delay Shocks

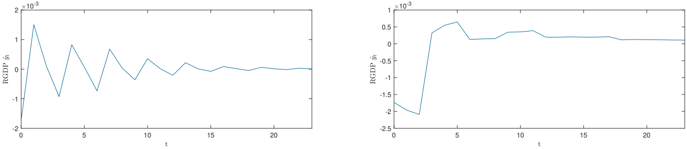
- Consider a $T-$period delay shock in sector i

$$\hat{X}_{i\tau} = -\varepsilon \text{ for } \tau = 0, ..., T-1$$
$$\hat{X}_{i\tau} = +\varepsilon \text{ for } \tau = T, ..., 2T-1$$

- Plot the response of aggregate real GDP $y_t = \sum \overline{p}_i \alpha_i y_{it}$
  - -1% of deliveries for 1 and 3 months

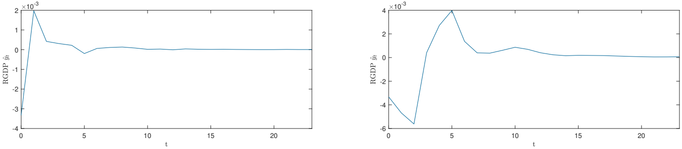
Delay Shocks IRFs

Figure 5: Nonferrous metal smelting and refining (bottleneck #2, $\tau = 3$ months)



(a) 1 month **(b)** 3 months

Figure 6: Plastics material and resin manuf. (bottleneck #3, $\tau = 5$ months)



(a) 1 month **(b)** 3 months

Echoes and Endogenous Fluctuations

VAR(1) Representation

- The *VAR* (τ_{max}) system can be put into VAR(1) form

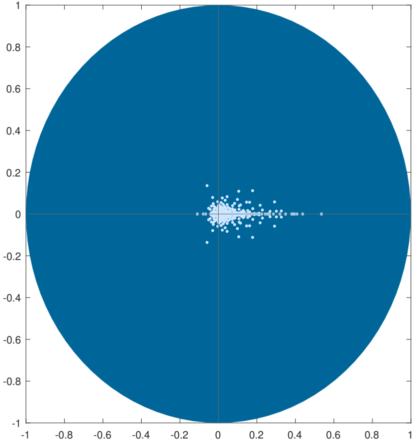
$$\underbrace{\begin{pmatrix} \hat{\mathbf{y}}_t \\ \hat{\mathbf{y}}_{t-1} \\ \\ \hat{\mathbf{y}}_{t-\tau_{max}+1} \end{pmatrix}}_{\equiv \mathbf{Y}_t} = \underbrace{\begin{pmatrix} \boldsymbol{\Omega}_1 & \boldsymbol{\Omega}_2 & \dots & & \boldsymbol{\Omega}_{\tau_{max}} \\ I_n & & & & \\ & I_n & & & \\ & & \ddots & & \\ & & & I_n & \end{pmatrix}}_{\equiv \mathbb{O}} \underbrace{\begin{pmatrix} \hat{\mathbf{y}}_{t-1} \\ \hat{\mathbf{y}}_{t-2} \\ \\ \hat{\mathbf{y}}_{t-\tau_{max}} \end{pmatrix}}_{\equiv \mathbf{Y}_{t-1}} + \underbrace{\begin{pmatrix} \hat{\mathbf{A}}_t \\ 0 \\ \vdots \\ 0 \end{pmatrix}}_{\equiv \mathbf{e}_t}$$

- *VAR* (1) representation:

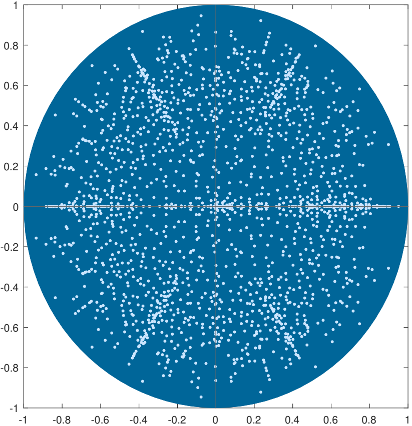
$$\mathbf{Y}_t = \mathbb{O}\mathbf{Y}_{t-1} + \mathbf{e}_t$$

- The system can oscillate if $\mathbb{O}$ has complex eigenvalues
  - Only true with time-to-build
  - In roundabout case, oscillations absent because collapsed within period

Spectrum

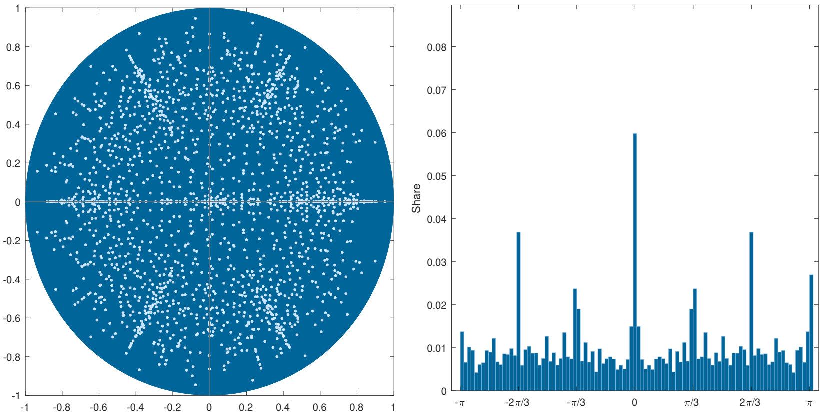


(c) LP83



(d) Heterogeneous T2B (Compustat)

Frequencies with Heterogeneous T2B



(a) Spectrum (T2B Compustat)

(b) Angular Frequency ω

$\Rightarrow$ Rich spectrum with peaks at periods of 2, 3 and 6 months

- Period $= \frac{1}{f} = \frac{2\pi}{\omega}$

Oscillations and Network Cycles

- Oscillations are a consequence of cycles (loops) in the network
- A simple result:

Proposition
A purely downstream production network (i.e. acyclical) displays no oscillations.

Proof.

- ► *There exists an ordering of sectors in which Ω is lower triangular with 0s on the diagonal*
- ► *All eigenvalues are 0*
- ► *Note: shocks vanish after a finite number of iterations (at most $N \times \tau_{max}$)*

☐

- Eigenvalues in the general case are too complicated
  - ► Algebraic graph theory: at most characterize 1st and 2nd largest eigenvalues...
  - ► ... but we can characterize the **Fourier spectrum**!

Refresher: Discrete Time Fourier Transform (DTFT)

- Any discrete-time 0-mean stationary process x_t can be represented by

$$x_t = \int_{-\pi}^{\pi} \delta\left(\omega\right) e^{i\omega t} d\omega$$

where $E\left[\delta\left(\omega\right)\right] = 0,\ E\left[\delta\left(\omega\right)\delta\left(\omega'\right)\right] = 0$ for $\omega \neq \omega'$

- The Discrete Time Fourier Transform (DTFT) is

$$\delta\left(\omega\right) = \frac{1}{2\pi} \sum_{t=-\infty}^{\infty} x_t e^{-i\omega t}$$

- The spectral density is

$$f\left(\omega\right) \equiv E\left[\delta\left(\omega\right)\overline{\delta\left(\omega\right)}\right]$$

DTFT and Autocorrelation Function

- Autocorrelation function (ACF)

$$\gamma_k = E\left[x_t x_{t-k}\right] \text{ for } k = -\infty, ..., \infty$$

- **Key property**: Fourier spectrum is the DTFT of the ACF

$$f(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \gamma_k e^{-i\omega k}$$

⇒ The ACF can be characterized analytically & using network topology

ACMF of a VAR(1)

- Recall the VAR(1) representation

$$\mathbf{Y}_t = \mathbb{O}\mathbf{Y}_{t-1} + \mathbf{e}_t$$

  and $\mathbf{\Sigma} = E\left[\mathbf{e}\mathbf{e}'\right]$ and $\mathbf{e}$ *iid*

- The Autocovariance Matrix Function $\mathbf{\Gamma}_k = E\left[\mathbf{Y}_t\mathbf{Y}'_{t-k}\right]$ is

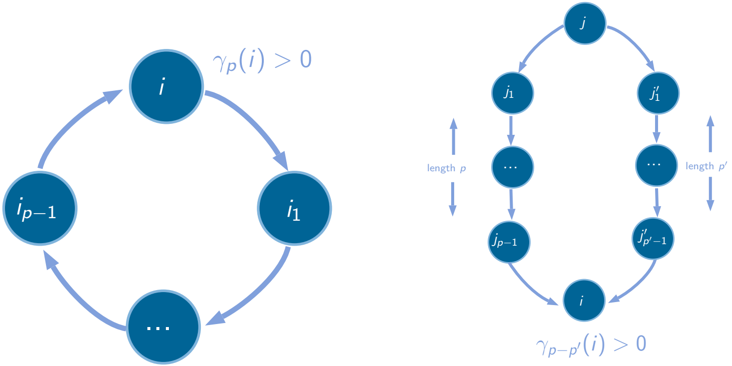
$$\mathbf{\Gamma}_0 = \sum_{k=0}^{\infty} \mathbb{O}^k \mathbf{\Sigma} \left(\mathbb{O}'\right)^k$$

$$\mathbf{\Gamma}_k = \mathbb{O}^k \mathbf{\Gamma}_0$$

- We can extract the relevant $\gamma_k\left(i\right) = E\left[\hat{y}_{it}\hat{y}_{it-k}\right]$ and construct spectrum
  - ... but provides little understanding

Sources of Serial Correlation

Serial correlation for sector i happens for only 2 reasons:

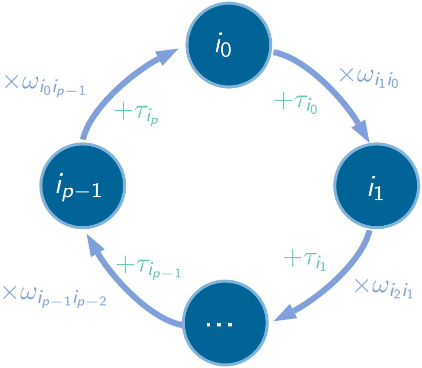


(a) Directed cycle

(b) Undirected cycle

⇒ shocks **echoe** in the production network through **cycles**

Directed Cycles



$p-$cycle $\varsigma = \left(i_0, i_1, \dots, i_{p-1}, i_p = i_0\right)$

- Duration of cycle:
  - $\tau\left(\varsigma\right) = \sum_{k=0}^{p-1} \tau_k$
- Weight of cycle:
  - $w\left(\varsigma\right) = \prod_{k=0}^{p-1} \omega_{i_{k+1} i_k}$

Cycles and Spectrum

Proposition

A $p-$cycle $\varsigma = (i_0, i_1, \ldots, i_{p-1}, i_p = i_0)$ contributes (at least) to the ACF

$$\gamma_{k\tau(\varsigma)}(i_0) = w(\varsigma)^k \sigma^2(\hat{y}_{i_0 t})$$

for $k = 1, \ldots, \infty$ and to the Fourier spectrum

$$f_{i_0}(\omega) = \frac{\sigma^2(\hat{y}_{i_0 t})}{2\pi}\left(2 + \frac{1 - w(\varsigma)^2}{1 + w(\varsigma)^2 - 2w(\varsigma)\cos(\omega\tau(\varsigma))}\right).$$

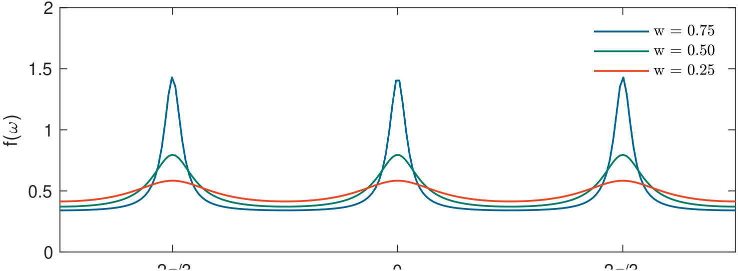


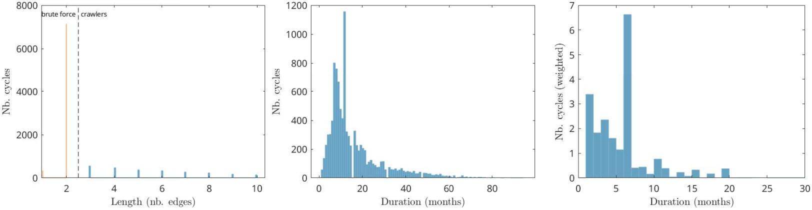
Figure 7: Spectrum of a cycle of duration $\tau = 3$ for different weights

▸ Poisson

Identifying Cycles

- Finding cycles in a network is a highly combinatorial problem
    - Cannot by brute force for length > 2-3
- We use a population of crawlers that travel the network randomly
    - Record cycles, their weights and durations whenever encountered
    - Not exhaustive, but cycles of length > 3 have low weights

Cycles (BEA I/O, Compustat)



(a) By length

(b) By duration

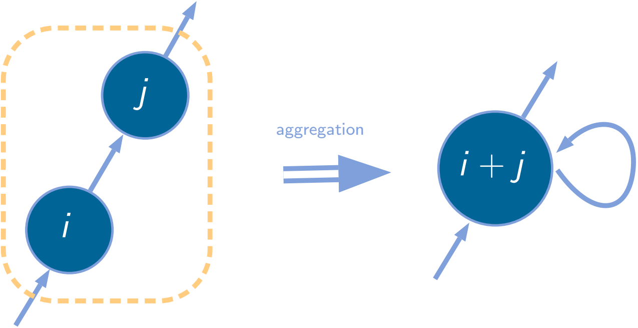
(c) By duration (weighted)

Top-3 Cycles (by weight, all cycles)

1. Cycle 47 - 47 (length 1)
    - Sectors:
        - Nonferrous metal (except aluminum) smelting and refining (331410)
    - Weight = 0.43, duration = 3
2. Cycle 8 - 8 (length 1)
    - Sectors:
        - Beef cattle ranching and farming, including feedlots and dual-purpose ranching and farming (1121A0)
    - Weight = 0.41, duration = 6
3. Cycle 205 - 205 (length 1)
    - Sectors:
        - Distilleries (312140)
    - Weight = 0.39, duration = 6

Aggregation Bias

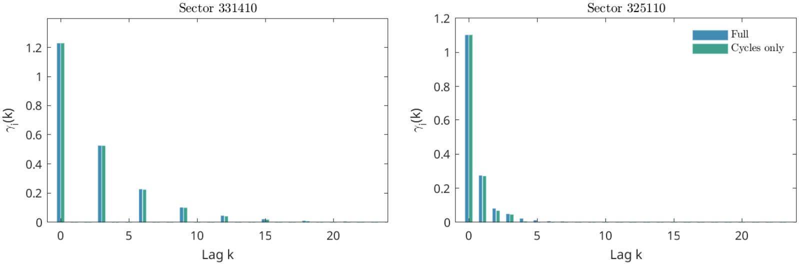
- BEA I/O tables display large self-loops on the diagonal
  - ⇒ Possibly spurious loops by aggregation (even at 6-digit level!)



Top-3 Cycles (by weight, length$\geq$2)

1. Cycle 229 - 233 - 229 (length 2)
    - Sectors:
        - Petrochemical manufacturing (325110)
        - Other basic organic chemical manufacturing (325190)
    - Weight = 0.03, duration = 3
2. Cycle 91 - 141 - 91 (length 2)
    - Sectors:
        - Other engine equipment manufacturing (333618)
        - Motor vehicle gasoline engine and engine parts manufacturing (336310)
    - Weight = 0.01, duration = 14
3. Cycle 217 - 218 - 217 (length 2)
    - Sectors:
        - Paperboard mills (322130)
        - Paperboard container manufacturing (322210)
    - Weight = 0.01, duration = 8

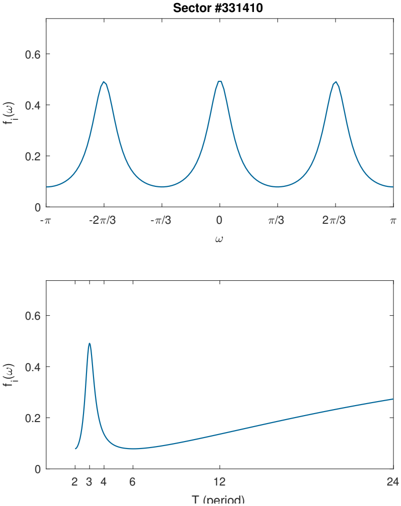
ACF Full vs. Directed Cycles only



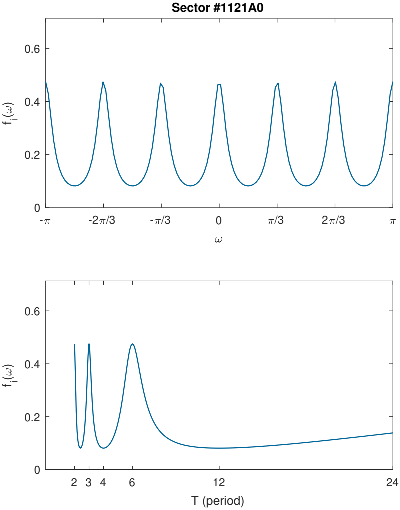
(a) Nonferrous metal (top 1-cycle) **(b)** Petrochemical manufacturing (top 2-cycle)

- Directed cycles account for virtually all the ACF
  - $R^2 = 0.9995$

Fourier Spectrum (full)



(a) Nonferrous metal (top 1-cycle #1)



(b) Beef cattle ranching and farming (top 1-cycle #2)

Sectoral Comovements and Aggregation

Dynamic Sectoral Comovements

- I/O linkages and time-to-build generate specific comovements
  - Across sectors
  - Over time
- Dynamic sectoral comovements are complex:

$$E\left[\hat{y}_{it}\hat{y}_{jt}\right] = [\mathbf{\Gamma_0}]_{ij} = \left[\sum_{\tau=0}^{\infty} \mathbb{O}^{\tau}\mathbf{\Sigma}\left(\mathbb{O}'\right)^{\tau}\right]_{ij}$$

$$E\left[\hat{y}_{it}\hat{y}_{jt-l}\right] = [\mathbf{\Gamma_l}]_{ij} = \left[\mathbb{O}^{l}\mathbf{\Gamma}_0\right]_{ij}$$

Unpacking Sectoral Comovements (1)

- Contemporaneous correlation:

$$
\begin{aligned}
E\left[\hat{y}_{it}\hat{y}_{jt}\right] = \left[\boldsymbol{\Gamma}_0\right]_{ij} &= \left[\sum_{\tau=0}^{\infty} \mathbb{O}^{\tau}\boldsymbol{\Sigma}\left(\mathbb{O}'\right)^{\tau}\right]_{ij} \\
&= \sum_{\tau=0}^{\infty}\sum_{k=1}^{N} \sum_{\substack{\text{walks } \varsigma_{k\to i},\varsigma_{k\to j} \\ \text{of duration } \tau}} w\left(\varsigma_{k\to i}\right) \times \sigma^2\left(\hat{A}_k\right) \times w\left(\varsigma_{k\to j}\right)
\end{aligned}
$$

Unpacking Sectoral Comovements (2)

- Lagged correlation:

$$E\left[\hat{y}_{it}\hat{y}_{jt-l}\right] = [\mathbf{\Gamma_l}]_{ij} = \left[\mathbb{O}^k \mathbf{\Gamma}_0\right]_{ij}$$

$$= \sum_{\tau=0}^{\infty}\sum_{k=1}^{N} \sum_{\substack{\varsigma_{k\to i} \text{ of duration } \tau+l \\ \varsigma_{k\to j} \text{ of duration } \tau}} w\left(\varsigma_{k\to i}\right) \times \sigma^2\left(\hat{A}_k\right) \times w\left(\varsigma_{k\to j}\right)$$

  - Dynamic comovements can be decomposed into dominant paths
    - **TO DO:** Use crawlers to parse the network and identify them

- We now illustrate those comovements with multi-sector IRFs

Multi-sector IRFs and GDP

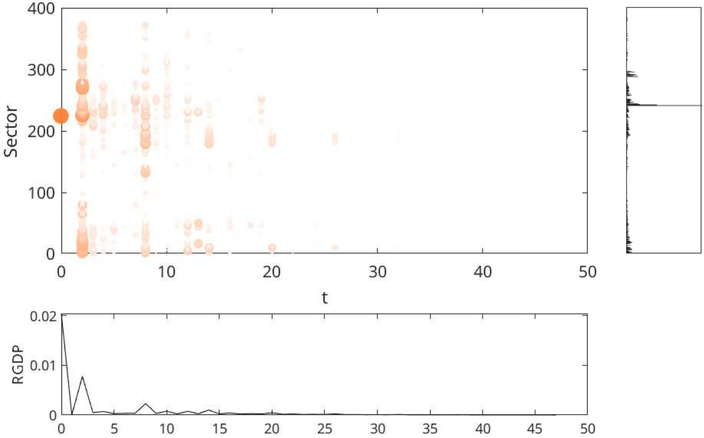


Figure 8: Sector 225 - Petroleum refineries (high Domar weight)

Multi-sector IRFs and Aggregates

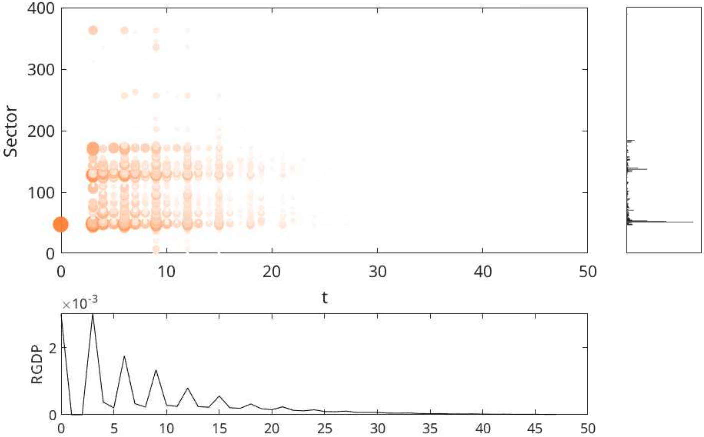


Figure 9: Sector 47 - Nonferrous metal (top 1-cycle)

Aggregation

- Oscillations survive aggregation
  - Large networks cycles appear in conditional GDP response
  - Depends how sectoral shocks spread to other sectors and involve other cycles/paths

- Real GDP $y_t = \sum \overline{p}_i \alpha_i y_{it}$ has ACF

$$\begin{aligned} E\left[\hat{y}_t \hat{y}_{t-k}\right] &= E\left[\boldsymbol{\mu}' \hat{\mathbf{y}}_t \hat{\mathbf{y}}'_{t-k} \boldsymbol{\mu}\right] \\ &= \boldsymbol{\mu}' \boldsymbol{\Gamma}_{\mathbf{k}} \boldsymbol{\mu} \end{aligned}$$

where $\mu_i = \overline{p}_i \alpha_i \overline{y}_i / \sum_j \overline{p}_j \alpha_j \overline{y}_j$

Spectrum of GDP

Proposition
The spectrum of real GDP is given by

$$f_y(\omega) = \underbrace{\sum_{i=1}^{N} \mu_i^2 f_i(\omega)}_{\textit{sum of sectoral spectra}} + \underbrace{\frac{1}{2\pi} \sum_{i \neq j} \sum_{k} \mu_i \mu_j \left[\boldsymbol{\Gamma}_k\right]_{ij} e^{-i\omega k}}_{\textit{sectoral comovement term}}$$

- The spectrum of GDP is the sum of two terms:
  - Sum of individual sectoral spectra implied by dominant cycles
  - Sum of spectra implied by sectoral comovements due to dominant paths

Spectrum of GDP

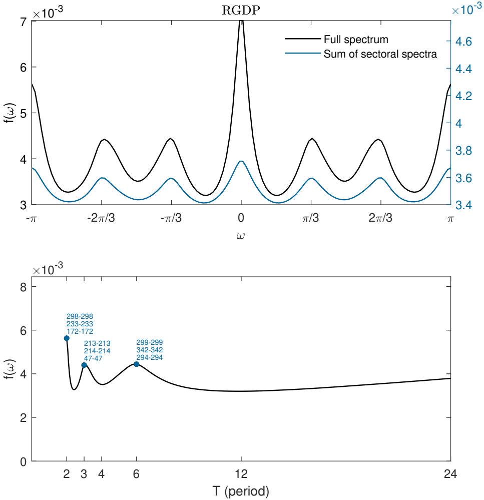


Figure 10: Spectrum of Real GDP

- Dominant 2-cycles
  - #298 Insurance carriers
  - #225 Petroleum refineries
  - #233 Organic chemical manuf.
- Dominant 3-cycles
  - #214 Leather and allied prod.
  - #213 Apparel manuf.
  - #43 Iron and steel mills
- Dominant 6-cycles
  - #299 Insurance, brokerage
  - #213 Hospitals
  - #14 Oil and gas

Empirical Evidence (in progress)

Empirical Challenges

- Data
  - Need high-frequency data (at least monthly) ⇒ price data?
  - Need non-distortionary detrending
    - Deflate prices by nominal wages
    - Large medium-term cycles ⇒ band-pass?
  - Spurious cycles in I/O tables
  - Other sources of serial correlation: sticky prices, capital, shocks...
- Theoretical
  - Model is simplistic and very particular
    - No inventory, no capital, constant expenditure shares, constant labor, only delivery lags...
  - Shocks are all iid to isolate internal propagation, some serial correlation may be needed

⇒ Need to design a proper way to evaluate the model's predictions

Price Time Series (BLS PPI 1947-2018)

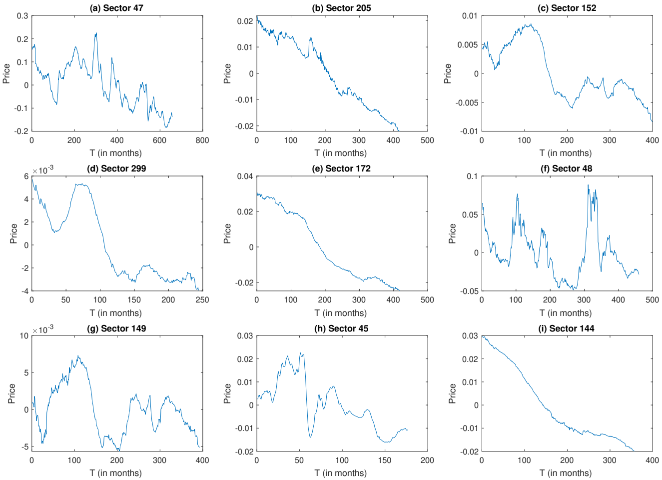
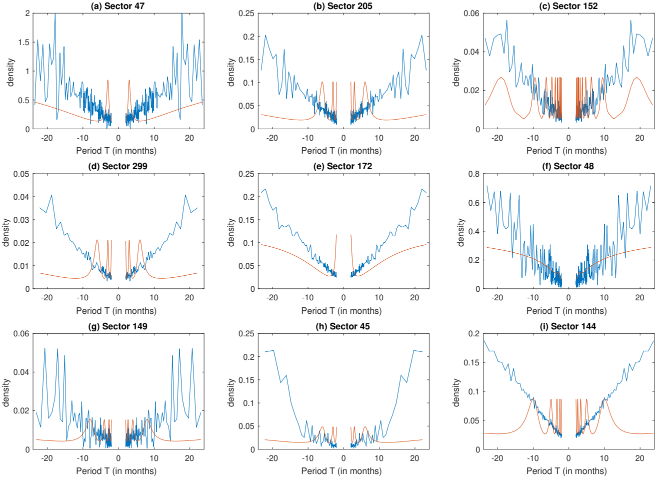


Figure 11: Price series for sectors with largest 1-cycle

Price Spectrum (BLS PPI 1947-2018)



(a) Sector 47

(b) Sector 205

(c) Sector 152

(d) Sector 299

(e) Sector 172

(f) Sector 48

(g) Sector 149

(h) Sector 45

(i) Sector 144

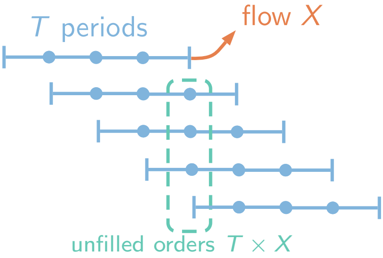
Figure 12: Price spectrum for sectors with largest 1-cycle

Conclusion

- Heterogeneous T2B significantly affects the propagation of shocks in network
  - Adds substantial & heterogeneous persistence across sectors
  - Can study impact of delay shocks & bottlenecks in time
- The economy fluctuates at frequencies implied by dominant cycles
  - Rich Fourier spectrum for aggregate GDP
- Complex dynamic sectoral comovements
  - Role of dominant paths to be further explored
- **Coming next:**
  - Empirical evidence
  - Robustness to inventories & other modeling assumptions

Backlog Ratio

- In steady state, backlog $= \frac{T \times X}{X} = T$



◂ Back

Poisson Model

- A common trick to model delays is to assume Poisson arrival:
  - For delivery lag τ, assume delivery with probability $1/\tau$ each period
- Example: suppose i_0 has a self-loop of weight w

$$\gamma_k\left(i_0\right) = w\left(1-\frac{1}{\tau}\right)^{k-1}\frac{1}{\tau}\,\sigma^2\left(\hat{y}_{i_0 t}\right) + \text{further iterations}$$

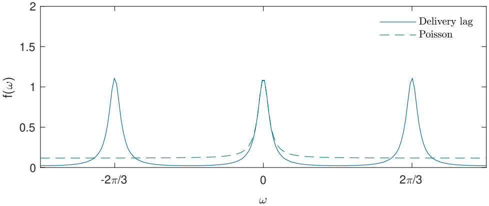


Figure 13: Spectrum of a Poisson model vs. delivery lag for $\tau = 3$

⇒ Poisson arrival heavily **distorts the spectrum**

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