Endogenous Production Networks Under Supply Chain Uncertainty

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How does uncertainty affect an economy's production network and, through that channel, macroeconomic aggregates?

Approach and results

We construct a model of endogenous network formation under uncertainty

- Firms create links with suppliers to acquire intermediate inputs
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- Network flexibility has large impact on welfare
- Sizable role for uncertainty during high-volatility events like the Great Recession

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Reduced-form evidence for the model mechanisms

- Links with riskier suppliers are more likely to be destroyed
- Riskier firms have lower Domar weights

Washington Post 15 November 2021

Economy

As supply lines strain, some corporations rewrite production playbook

Many firms are eyeing regional networks to replace globe-circling supply chains

Listen to article 8 min

Trucks line up to enter a Port of Oakland shipping terminal in Oakland, Calif. (Noah Berger/AP)

“As the disruptions persist, executives are embracing more lasting measures, moving production to new suppliers or different countries and relaxing their traditional fixation with low costs.”

Survey evidence

Surveys of business executives

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- Global survey of small and medium firms: 39% report that losing their main supplier would adversely affect their operation, and 14% report that they would need to significantly downsize their business, require emergency support or shut down (Zurich Insurance Group, 2015)
- COVID-19 pandemic: 70% agreed that the pandemic pushed companies to favor higher supply chain resiliency instead of purchasing from the lowest-cost supplier (Foley & Lardner, 2020)

Related literature

Uncertainty

- Bloom (2009); Fernandez-Villaverde et al (2011); Bloom (2014); Bloom et al (2018); and many others ...

Exogenous production networks

- Long and Plosser (1983); Dupor (1999); Horvath (2000); Acemoglu et al (2012); Carvalho and Gabaix (2013); and many others ...

Endogenous production networks

- Oberfield (2018); Acemoglu and Azar (2020); Taschereau-Dumouchel (2021); Acemoglu and Tahbaz-Salehi (2021); Ghassibe (2022) and many others ...

Model

Model

Static model with two types of agents

1. Representative household: owns the firms, supplies labor and consumes
2. Firms: produce differentiated goods using labor and intermediate inputs
    - There are n industries/goods, indexed by $i \in \{1, \ldots, n\}$
    - Representative firm that behaves competitively

Production technique

Each firm i has access to a set of production techniques $\mathcal{A}_i$.

A technique $\alpha_i \in \mathcal{A}_i$ specifies

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These techniques are Cobb-Douglas production functions

▸ ζ

- We identify $\alpha_i = (\alpha_{i1}, \ldots, \alpha_{in})$ with the input shares

$$F(\alpha_i, L_i, X_i) = e^{\varepsilon_i} \zeta(\alpha_i) A_i(\alpha_i) L_i^{1 - \sum_{j=1}^n \alpha_{ij}} \prod_{j=1}^n X_{ij}^{\alpha_{ij}},$$

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Allow adjustment along intensive and extensive margins: $\mathcal{A}_i = \left\{\alpha_i \in [0,1]^n : \sum_{j=1}^n \alpha_{ij} \leq \overline{\alpha}_i < 1\right\}$.

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Example: A car manufacturer can use only steel or only carbon fiber, or a combination of both.

Production technique

Assumption

$A_i(\alpha_i)$ is smooth and strictly log-concave.

Implication: There are ideal input shares α_{ij}° that maximize A_i

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Example

$$\log A_i(\alpha_i) = -\sum_{j=1}^{n} \kappa_{ij}\left(\alpha_{ij} - \alpha_{ij}^{\circ}\right)^2 - \kappa_{i0}\left(\sum_{j=1}^{n} \alpha_{ij} - \sum_{j=1}^{n} \alpha_{ij}^{\circ}\right)^2,$$

Source of uncertainty and timing

Firms are subject to productivity shocks $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n) \sim \mathcal{N}(\mu, \Sigma)$

- Vector μ captures optimism/pessimism about productivity
- Covariance matrix Σ captures uncertainty and correlations

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Timing

1. Before ε is realized: Production techniques are chosen
    - Beliefs (μ, Σ) affect technique choice $\rightarrow$ production network $\alpha \in \mathcal{A}$ is endogenous
2. After ε is realized: All other decisions are taken
   Only impact of uncertainty on decisions is through technique choice

▸ One tech.

Household

The representative household makes decisions after ε is realized

- Owns the firms
- Supplies one unit of labor inelastically
- Chooses *state-contingent* consumption $(C_1, \ldots, C_n)$ to maximize

$$u\left(\left(\frac{C_1}{\beta_1}\right)^{\beta_1} \times \cdots \times \left(\frac{C_n}{\beta_n}\right)^{\beta_n}\right),$$

subject to the *state-by-state* budget constraint

$$\sum_{i=1}^{n} P_i C_i \leq 1,$$

where u is CRRA with relative risk aversion $\rho \geq 1$.

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- We refer to aggregate consumption $Y = \prod_{i=1}^{n} \left(\beta_i^{-1} C_i\right)^{\beta_i}$ as GDP.

Household

Two key quantities from the household's problem

1. The stochastic discount factor of the household is

$$\Lambda = u'\left(Y\right)/\overline{P}$$

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2. log GDP as a function of prices

$$y = -\beta^{\top} p,$$

   where $y = \log Y$, $p = (\log P_1, \ldots, \log P_n)$ and $\beta = (\beta_1, \ldots, \beta_n)$.

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   $\Rightarrow$ We only need prices to compute GDP

Problem of the firm

Firms solve a two-stage problem

1. Before ε is drawn: Choose production technique α_i
    - ex ante decision under uncertainty
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Problem of the firm: Labor and intermediate inputs

For a given technique α_i, the cost minimization problem of the firm is

$$K_i(\alpha_i, P) := \min_{L_i, X_i} \left(L_i + \sum_{j=1}^{n} P_j X_{ij} \right), \text{ subject to } F(\alpha_i, L_i, X_i) \geq 1$$

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1. Constant returns to scale $\rightarrow K_i$ does not depend on firm size
2. Given that each technique is Cobb-Douglas,

$$K_i(\alpha_i, P) = \frac{1}{e^{\varepsilon_i} A_i(\alpha_i)} \prod_{j=1}^{n} P_j^{\alpha_{ij}}.$$

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Problem of the firm: Production technique

Firm i chooses a technique $\alpha_i \in \mathcal{A}_i$ to maximize discounted profits

$$\alpha_i^* \in \arg\max_{\alpha_i \in \mathcal{A}_i} \mathrm{E}\left[\Lambda Q_i \left(P_i - K_i\left(\alpha_i, P\right)\right)\right]$$

where Q_i is equilibrium demand for good i and Λ is the SDF.

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Lemma

In equilibrium, the technique choice of the representative firm in sector i solves

$$\alpha_i^* \in \arg\min_{\alpha_i \in \mathcal{A}_i} \mathrm{E}\left[k_i\left(\alpha_i, \alpha^*\right)\right] + \mathrm{Cov}\left[\lambda\left(\alpha^*\right), k_i\left(\alpha_i, \alpha^*\right)\right]. \tag{1}$$

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The firm prefers techniques with low

1. expected unit cost
2. unit cost when marg. utility is high $\rightarrow$ firm "inherits" the household's risk aversion through λ

We can expand the two terms to minimize

$$\mathrm{E}\left[k_i\left(\alpha_i, \alpha^*\right)\right] = -a_i\left(\alpha_i\right) + \sum_{j=1}^{n} \alpha_{ij}\,\mathrm{E}\left[p_j\right]$$

Firm prefers techniques with high TFP and low average input prices.

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$$\mathrm{Cov}\left[\lambda, k_i\right] = \sqrt{\mathrm{V}\left[\lambda\right]} \times \mathrm{Corr}\left[\lambda, k_i\right]\sqrt{\mathrm{V}\left[k_i\right]}$$

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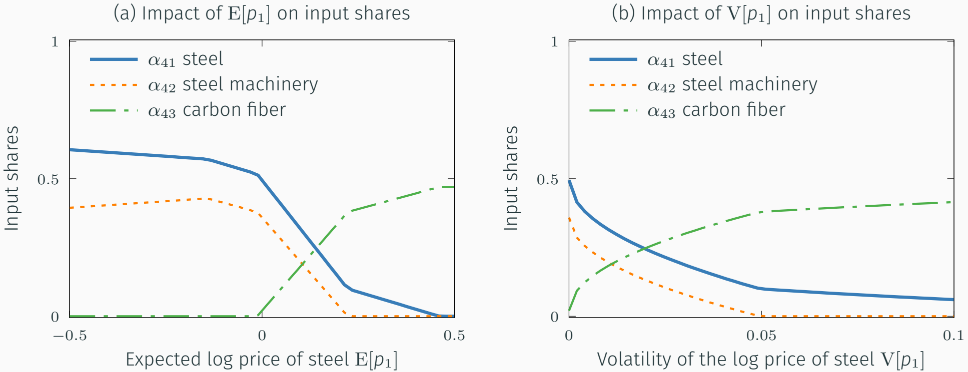
In general $\mathrm{Corr}\left[\lambda, k_i\right] > 0 \rightarrow$ Minimize variance of k_i

$$\mathrm{V}\left[k_i\right] = \text{cte} + \underbrace{\sum_{j=1}^{n} \alpha_{ij}^2\,\mathrm{V}\left[p_j\right]}_{\text{stable prices}} + \underbrace{\sum_{j \neq k} \alpha_{ij}\alpha_{ik}\,\mathrm{Cov}\left[p_j, p_k\right]}_{\text{uncorrelated prices}} + \underbrace{2\,\mathrm{Cov}\left[-\varepsilon_i, \sum_{j=1}^{n} \alpha_{ij} p_j\right]}_{\text{uncorrelated with own } \varepsilon_i}$$

Back to our car manufacturer example

- Firm $i = 4$ can use steel (input 1), steel milling machines (input 2) or carbon fiber (input 3)

$$a_4(\alpha_4) = -\sum_{j=1}^{4} \kappa_j \left(\alpha_{4j} - \alpha_{4j}^{\circ}\right)^2 - \psi_1 \left(\alpha_{41} - \alpha_{42}\right)^2 - \psi_2 \left(\left(\alpha_{41} + \alpha_{43}\right) - \left(\alpha_{41}^{\circ} + \alpha_{43}^{\circ}\right)\right)^2,$$



Definition

An equilibrium is a technique for every firm α^* and a stochastic tuple $(P^*, C^*, L^*, X^*, Q^*, \Lambda^*)$ such that

1. (Unit cost pricing) For each $i \in \{1, \ldots, n\}$, $P_i^* = K_i(\alpha_i^*, P^*)$.
2. (Optimal technique choice) For each $i \in \{1, \ldots, n\}$, factor demand L_i^* and X_i^*, and the technology choice $\alpha_i^* \in \mathcal{A}_i$ solves the firm's problem.
3. (Consumer maximization) The consumption vector C^* solves the household's problem.
4. (Market clearing) For each $i \in \{1, \ldots, n\}$,

$$Q_i^* = C_i^* + \sum_{j=1}^{n} X_{ji}^*,$$

$$Q_i^* = F_i(\alpha_i^*, L_i^*, X_i^*),$$

$$\sum_{i=1}^{n} L_i^* = 1.$$

Fixed-network economy

GDP in a fixed-network economy

Define a firm's Domar weight ω_i as its sales share

$$\omega_i(\alpha) := \frac{P_i Q_i}{PC}$$

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$$\omega_i(\alpha) := \frac{P_i Q_i}{PC} = \beta^\top \mathcal{L}(\alpha) 1_i$$

Domar weights depend on

1. Demand from the household through β
2. Demand from intermediate good producers through $\mathcal{L}(\alpha) = (I - \alpha)^{-1} = I + \alpha + \alpha^2 + \ldots$

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→ Domar weights are constant for a fixed network

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Lemma (Hulten's Theorem)

Under a given network α, the log of GDP $y = \log Y$ is given by

$$y = \omega(\alpha)^\top (\varepsilon + a(\alpha)).$$

Impact of beliefs on GDP

Proposition (Hulten's Theorem in expectation)

For a fixed network α,

1. The impact of μ_i on expected log GDP is given by
$$\frac{\partial \, \mathrm{E}[y]}{\partial \mu_i} = \omega_i.$$
2. The impact of Σ_{ij} on the variance of log GDP is given by
$$\frac{\partial \, \mathrm{V}[y]}{\partial \Sigma_{ij}} = \omega_i \omega_j.$$
3. μ does not affect $\mathrm{V}[y]$ and Σ does not affect $\mathrm{E}[y]$.

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For a fixed network

1. Domar weights ω are enough to understand log GDP
2. Since $\omega_i > 0$ shocks have intuitive impact.

Flexible-network economy

Equilibrium and efficiency

The economy is fully competitive and undistorted by frictions or externalities.

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Proposition

There exists a unique equilibrium and it is efficient. The equilibrium network solves

$$\mathcal{W} := \max_{\alpha \in \mathcal{A}} \mathrm{E}\left[y\left(\alpha\right)\right] - \frac{1}{2}\left(\rho - 1\right) \mathrm{V}\left[y\left(\alpha\right)\right]$$

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Implications

1. The planner prefers networks that balance high $\mathrm{E}\left[y\left(\alpha\right)\right]$ with low $\mathrm{V}\left[y\left(\alpha\right)\right]$
2. Complicated network formation problem $\rightarrow$ simpler optimization problem.

Recasting the planner's problem in the space of Domar weights

We can write the planner's problem as

$$\max_{\alpha \in \mathcal{A}} \underbrace{\omega(\alpha)^\top (\mu + a(\alpha))}_{\mathrm{E}[y(\alpha)]} - \frac{1}{2}(\rho - 1) \underbrace{\omega(\alpha)^\top \Sigma \omega(\alpha)}_{\mathrm{V}[y(\alpha)]}$$

This problem depends almost exclusively on ω, except for $a(\alpha)$...

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Define the aggregate TFP shifter $\bar{a}(\omega)$ as

$$\bar{a}(\omega) := \max_{\alpha \in \mathcal{A}} \omega^\top a(\alpha)$$

Recasting the planner's problem in the space of Domar weights

We can write the planner's problem as

$$\max_{\alpha\in\mathcal{A}} \underbrace{\omega(\alpha)^\top (\mu + a(\alpha))}_{\mathrm{E}[y(\alpha)]} - \frac{1}{2}(\rho-1)\underbrace{\omega(\alpha)^\top \Sigma \omega(\alpha)}_{\mathrm{V}[y(\alpha)]}$$

This problem depends almost exclusively on ω, except for $a(\alpha)$...

Multiple networks α correspond to a Domar weight vector ω. Which one is the best?

Define the aggregate TFP shifter $\bar{a}(\omega)$ as

$$\bar{a}(\omega) := \max_{\alpha\in\mathcal{A}} \omega^\top a(\alpha)$$

Recast the planner's problem in the space of Domar weights

$$\mathcal{W} = \max_{\omega\in\mathcal{O}} \underbrace{\omega^\top \mu + \bar{a}(\omega)}_{\mathrm{E}[y]} - \frac{1}{2}(\rho-1)\underbrace{\omega^\top \Sigma \omega}_{\mathrm{V}[y]}$$

Beliefs and the production network

Impact of beliefs on the network

Domar weights are constant when the network is fixed. But when it is flexible...

Proposition

The Domar weight ω_i of firm i is increasing in μ_i and decreasing in Σ_{ii}.

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1. Equilibrium: Firms rely more on high-μ_i and low-Σ_{ii} firms as suppliers.
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Flexible network $\rightarrow$ beneficial changes are amplified while adverse changes are mitigated.

▸ Data ▸ Impact on α

Impact of beliefs on the network

What about the impact of μ_i on the Domar weight of other sectors $j \neq i$?

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Risk-adjusted productivity $\mathcal{E}$: measure of how higher exposure to ε affects the household's utility

$$\mathcal{E} = \underbrace{\mu}_{\mathrm{E}[\varepsilon]} - \underbrace{(\rho - 1)\,\Sigma\omega}_{\mathrm{Cov}[\varepsilon,\lambda]}$$

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The impact of a change in beliefs (μ, Σ) can be summarized by its direct impact on $\mathcal{E}$

$$\frac{\partial \mathcal{E}}{\partial \mu_i} = \mathbf{1}_i, \text{ and } \frac{\partial \mathcal{E}}{\partial \Sigma_{ij}} = -\frac{1}{2}\,(\rho - 1)\,(\omega_j \mathbf{1}_i + \omega_i \mathbf{1}_j)$$

Impact of beliefs on the network

Proposition

Let γ denote either μ_i or Σ_{ij}. If $\omega \in \text{int}\,\mathcal{O}$, then

$$\frac{d\omega}{d\gamma} = \underbrace{-\mathcal{H}^{-1}}_{\text{propagation}} \times \underbrace{\frac{\partial \mathcal{E}}{\partial \gamma}}_{\text{impulse}},$$

where $\mathcal{H}$ is an $n \times n$ matrix.

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The propagation matrix $\mathcal{H}^{-1}$ captures global substitution patterns across sectors

Impact of a beneficial change to i (higher $\mathcal{E}_i$) on ω_j

1. If $\mathcal{H}_{ij}^{-1} < 0$, i and j are complements $\implies$ ω_j increases
2. If $\mathcal{H}_{ij}^{-1} > 0$, i and j are substitutes $\implies$ ω_j decreases

Determinant of substitution patterns

$$\mathcal{H} = \nabla^2 \bar{a} - \underbrace{(\rho - 1)\,\Sigma}_{\frac{d\,\mathrm{Cov}[\varepsilon,\lambda]}{d\omega}}$$

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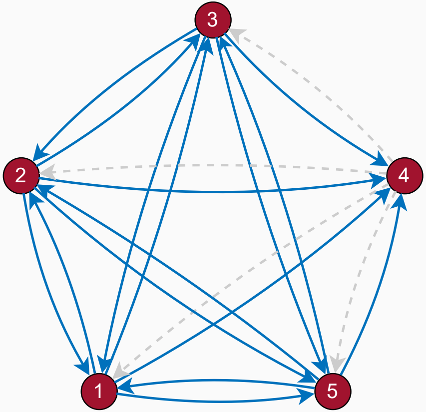
1. Aggregate TFP shifter function $\bar{a}$
   - Local substitution patterns in $(a_1, \ldots, a_n)$ contribute to global substitution patterns

2. Covariance matrix Σ
   - If Σ_{ij} is larger the planner wants to lower ω_j after an increase in ω_i to reduce aggregate risk.

$$\frac{\partial \mathcal{H}_{ij}^{-1}}{\partial \Sigma_{ij}} > 0$$

Example: Impact of beliefs on the network

Simple example of possible substitution patterns



Baseline

▸ Details

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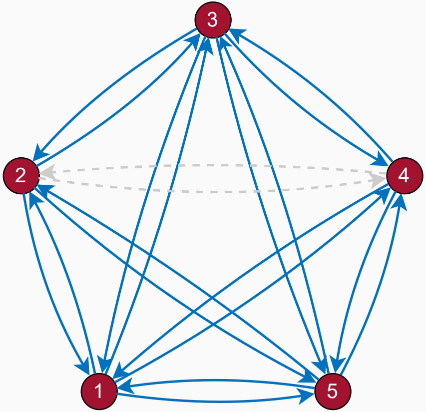
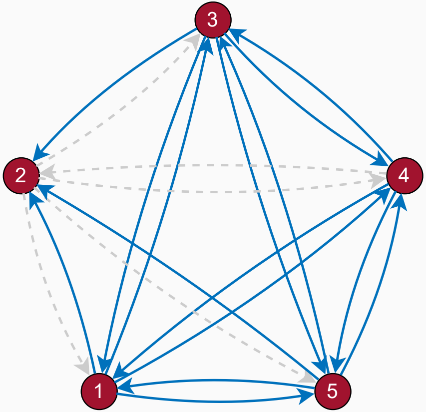


Small increase in $\Sigma_{22} \rightarrow$ Firms also purchase from 4 to diversify

▸ Details

Example: Impact of beliefs on the network

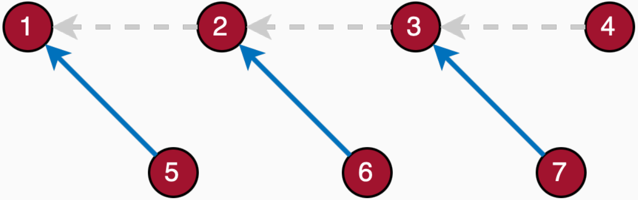
Simple example of possible substitution patterns



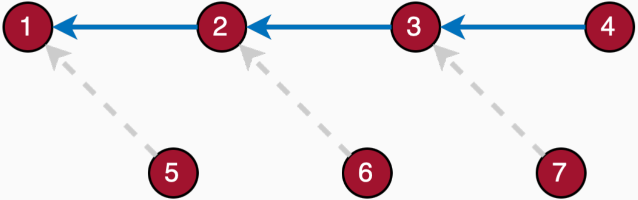
Large increase in Σ_{22} $\rightarrow$ Firms drop 2 as a supplier

▶ Details

Example: Cascading effect of uncertainty



(a) High uncertainty about ε_4



(b) Low uncertainty about ε_4

Beliefs and welfare

Impact of beliefs on welfare

Let γ denote either μ_i or Σ_{ij} and let $W(\alpha, \mu, \Sigma)$ be welfare under α.

$$\frac{d\mathcal{W}}{d\gamma} = \underbrace{\frac{dW}{d\alpha}}_{\substack{\text{Impact} \\ \textit{of}\text{ network}}} \times \underbrace{\frac{d\alpha}{d\gamma}}_{\substack{\text{Impact} \\ \textit{on}\text{ network}}} + \underbrace{\frac{\partial W}{\partial \gamma}}_{\substack{\text{Fixed network} \\ \text{effect}}}$$

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Proposition

Let γ denote either μ_i or Σ_{ij}. Welfare responds to a marginal change in γ *as if* the network were fixed at its equilibrium value α^*, that is

$$\frac{d\mathcal{W}(\mu, \Sigma)}{d\gamma} = \frac{\partial W(\alpha^*, \mu, \Sigma)}{\partial \gamma}.$$

Impact of beliefs on welfare

What about non-marginal changes in beliefs?

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Corollary

Let $\alpha^*(\mu, \Sigma)$ be the equilibrium network. A change in beliefs from (μ, Σ) to (μ', Σ') implies

$$\underbrace{\mathcal{W}(\mu', \Sigma') - \mathcal{W}(\mu, \Sigma)}_{\text{Change under a flexible network}} \geq \underbrace{W(\alpha^*(\mu, \Sigma), \mu', \Sigma') - W(\alpha^*(\mu, \Sigma), \mu, \Sigma)}_{\text{Change under a fixed network}}.$$

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Under a flexible network the planner has more tools to maximize welfare.

$\Rightarrow$ Changes that are beneficial are amplified. Changes that are detrimental are dampened.

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Since we know how welfare behaves under a fixed network...

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Corollary

The impact of μ_i on welfare is given by

$$\frac{d\mathcal{W}}{d\mu_i} = \frac{\partial W}{\partial \mu_i} = \omega_i,$$

and the impact of Σ_{ij} on welfare is given by

$$\frac{d\mathcal{W}}{d\Sigma_{ij}} = \frac{\partial W}{\partial \Sigma_{ij}} = -\frac{1}{2}\left(\rho - 1\right)\omega_i \omega_j$$

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- Higher $\mu \implies$ firms are more productive on average $\implies$ higher welfare
- Higher correlation or uncertainty $\implies$ more aggregate risk $\implies$ lower welfare

Beliefs and GDP

Effect of uncertainty on GDP

Proposition

Uncertainty lowers expected GDP, in the sense that $\mathrm{E}[y]$ is largest when $\Sigma = 0$.

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Uncertainty lowers expected GDP, in the sense that $\mathrm{E}[y]$ is largest when $\Sigma = 0$.

Intuition

1. Equilibrium: With uncertainty, firms seek stability at the cost of expected productivity.
2. Planner: Only objective is to maximize $\mathrm{E}[y]$.

$$\mathcal{W} := \max_{\alpha \in \mathcal{A}} \mathrm{E}[y(\alpha)] - \cancel{\frac{1}{2}(\rho - 1)\mathrm{V}[y(\alpha)]}$$

Impact of a *marginal* change in beliefs

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Impact of a *marginal* change in beliefs

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If the resp. of $\mathrm{E}\left[y\right]$ is better than under a fixed network, then the resp. of $\mathrm{V}\left[y\right]$ must be worse.

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If the resp. of $\mathrm{E}[y]$ is better than under a fixed network, then the resp. of $\mathrm{V}[y]$ must be worse.

Corollary

Without uncertainty ($\Sigma = 0$) Hulten's theorem holds, such that $\frac{d\,\mathrm{E}[y]}{d\mu_i} = \omega_i$.

Without uncertainty $\mathrm{E}[y]$ responds *as if* the network were fixed!

Impact of a marginal change in beliefs

Lemma

If $\omega \in \text{int}\, \mathcal{O}$, that some condition on Σ holds and that all sectors are global substitutes, then

$$\frac{d\,\mathrm{E}[y]}{d\mu_i} < \omega_i, \quad \text{and} \quad \frac{d\,\mathrm{V}[y]}{d\mu_i} < 0.$$

and

$$\frac{d\,\mathrm{E}[y]}{d\Sigma_{ij}} > 0, \quad \text{and} \quad \frac{d\,\mathrm{V}[y]}{d\Sigma_{ij}} > \omega_i\omega_j,$$

where the right-hand side of these inequalities is the fixed-network effect.

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1. Increase in $\mu_i \implies \omega_i$ increases
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3. If Σ_{jj} is large relative to Σ_{ii} than the variance of GDP $\mathrm{V}[y]$ decreases
4. By our earlier result, $\mathrm{E}[y]$ must grow by less than under a fixed network (ω_i)

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Under global complementarity the inequalities are reversed

Example: Counterintuitive impact of a change in (μ, Σ)

Under some conditions, an increase in μ_i can lead to a decline in $\mathrm{E}[y]$

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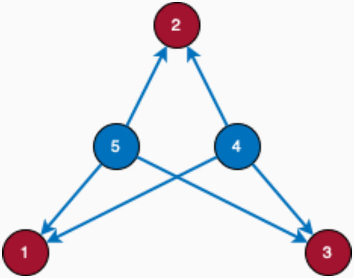

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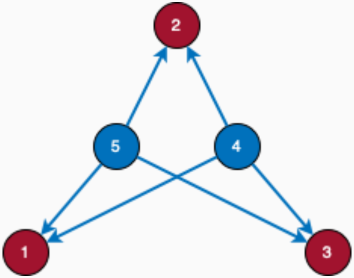

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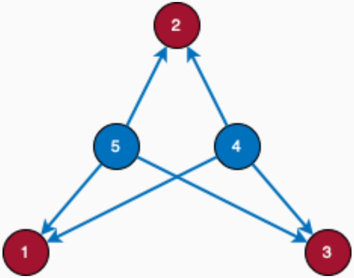

- Firms 4 and 5 are global substitutes through local substitution in a_1, a_2 and a_3
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- Firm 5 is safe (low Σ_{55}) but unproductive (low μ_5)

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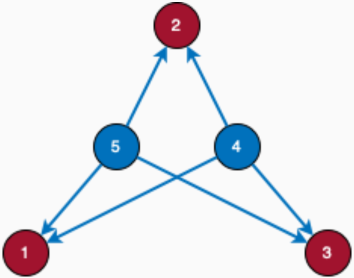
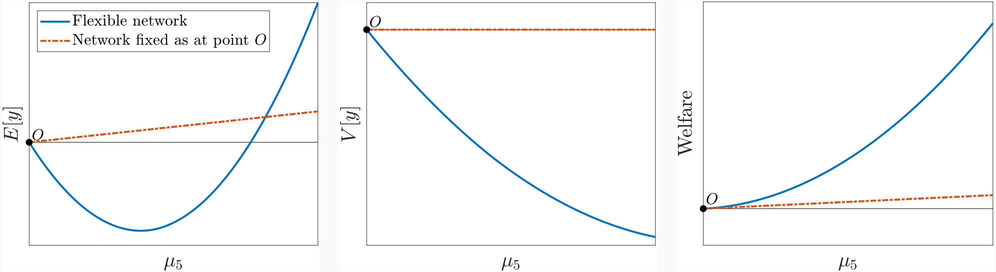


- Firms 4 and 5 are global substitutes through local substitution in a_1, a_2 and a_3
- Firm 4 is risky (high Σ_{44}) but productive (high μ_4)
- Firm 5 is safe (low Σ_{55}) but unproductive (low μ_5)
- Increase μ_5: Move away from high-μ firm 4 toward low-μ firm 5 $\Rightarrow$ $\mathrm{E}[y]$ falls

Example: Counterintuitive impact of a change in (μ, Σ)



(a) $\mathrm{E}[y]$ as a function of μ_5 (b) $\mathrm{V}[y]$ as a function of μ_5 (c) Welfare as a function of μ_5

Quantitative exploration

Calibration

Data

- Annual United States data from 1947 to 2020 about 37 sectors

Calibration

- Consumption shares β and ideal shares α° taken from the data
- Risk-aversion ρ and cost of deviating κ are **estimated**
- ε_t is random walk with drift and time-varying uncertainty and is **estimated**

▸ Data details ▸ Estimation details

Calibrated economy

Estimated risk aversion: $\rho = 4.27$

Calibrated economy

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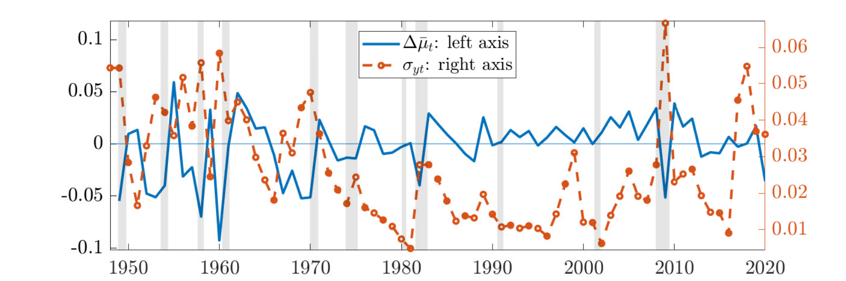
Estimated evolution of beliefs

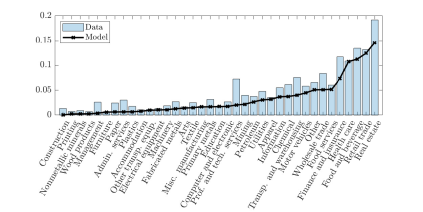


$$\Delta\bar{\mu}_t = \sum_{j=1}^{n} \omega_{jt}\Delta\mu_{jt} \text{ and } \sigma_{yt} = \sqrt{\mathrm{V}[y]} = \sqrt{\omega_t'\Sigma_t\omega_t}.$$

▸ κ's

Calibrated economy: Domar weights

The calibrated Domar weights fit the data reasonably well



Beliefs have the expected impact on Domar weights

	Statistic	Data	Model
(1)	Average Domar weight $\bar{\omega}_j$	0.047	0.032
(2)	Standard deviation $\sigma(\omega_j)$	0.0050	0.0021
(3)	Coefficient of variation $\sigma(\omega_j)/\bar{\omega}_j$	0.11	0.07
(4)	$\text{Corr}(\omega_{jt}, \mu_{jt})$	0.08	0.08
(5)	$\text{Corr}(\omega_{jt}, \Sigma_{jjt})$	−0.37	−0.31

Isolating the mechanism

Two useful counterfactuals

1. Fixed-network economy
    - No change in network $\rightarrow$ capture the full effect of network adjustments
2. “*as if* $\Sigma = 0$” economy
    - Uncertainty has no impact on network $\rightarrow$ capture the impact of uncertainty
    - Recall: only impact of uncertainty on expected GDP is through the network

Isolating the mechanism

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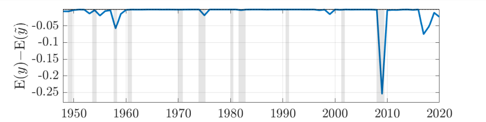
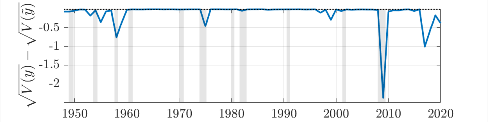
	Baseline model compared to...	
	Fixed network	As if $\Sigma = 0$
Expected GDP $\mathbb{E}[y(\alpha)]$	$+2.122\%$	-0.008%
Std. dev. of GDP $\sqrt{\mathbb{V}[y(\alpha)]}$	$+0.131\%$	-0.105%
Welfare $\mathcal{W}$	$+2.109\%$	$+0.010\%$

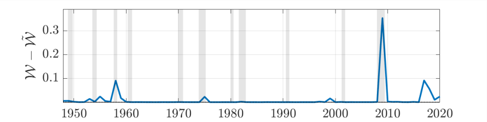
Calibrated model vs As if $\Sigma = 0$ alternative



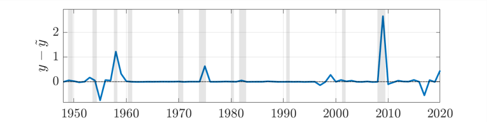
(a) Difference in expected GDP



(b) Difference in expected std. dev. of GDP



(c) Difference in expected welfare



(d) Difference in realized GDP

- During periods of high volatility, uncertainty matters.

Reduced-form evidence for the model mechanisms

Links with riskier suppliers are more likely to be destroyed

Use detailed U.S. data on firm-to-firm relationship (Factset 2003–2016)

Regress a dummy for link destruction on supplier uncertainty measures

- Instruments from Alfaro, Bloom and Lin (2019)

▸ Details

	Dummy for last year of supply relationship		
	(1) OLS	(2) IV	(3) IV
ΔVol_{t-1} of supp.	0.026**	0.097***	0.1494**
	(0.010)	(0.029)	(0.064)
1st moment of IVs	No	Yes	Yes
Type of volatility	Realized	Realized	Implied
Fixed effects	Yes	Yes	Yes
Observations	35,629	35,620	26,195
F-statistic	—	39.0	23.2

All specifications include year × customer × supplier industry (2SIC) fixed effects. Standard errors are two-way clustered at the customer and the supplier levels. *F*-statistics are Kleibergen-Paap. *, **, *** indicate significance at the 10%, 5%, and 1% levels, respectively.

- Doubling volatility → 12 p.p. increase in probability link destroyed (IV)

Domar weights and uncertainty in the data

Firms with higher uncertainty have lower Domar weights, in line with the model

- Specifications, uncertainty measures and instruments from Alfaro, Bloom and Lin (2019)

	Change in Domar weight		
	(1) OLS	(2) IV	(3) IV
$\Delta \text{Volatility}_{i,t-1}$	-0.043^{***}	-0.250^{***}	-0.672^{***}
	(0.004)	(0.076)	(0.185)
1st moment of IVs	No	Yes	Yes
Type of volatility	Realized	Realized	Implied
Fixed effects	Yes	Yes	Yes
Observations	111,587	26,962	16,862
F-statistic	—	17.0	9.8

All specifications include year and firm fixed effects. Standard errors are clustered at the industry (3SIC) level. F-statistics are Kleibergen-Paap.
$^{*}, ^{**}, ^{***}$ indicate significance at the 10%, 5%, and 1% levels, respectively.

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Conclusion

Conclusion

Main contributions

- We construct a model in which beliefs, and in particular uncertainty, affect the production network.
- During periods of high uncertainty firms purchase from safer but less productive suppliers which leads to a decline in GDP.
- Mechanism might be quantitatively important during periods of high uncertainty.

Future research

- Use firm-level data to calibrate the model — firm-to-firm network is more sparse and links are often broken.
- Use the model to evaluate the impact of uncertainty on global supply chains.

Thank you!

More about the data

United States data from vom Lehn and Winberry (2021)

- Input-output tables, sectoral total factor productivity, consumption shares

Mining	Utilities	Construction
Wood products	Nonmetallic minerals	Primary metals
Fabricated metals	Machinery	Computer and electronic manuf.
Electrical equipment manufacturing	Motor vehicles manufacturing	Other transportation equipment
Furniture and related manufacturing	Misc. manufacturing	Food and beverage manufacturing
Textile manufacturing	Apparel manufacturing	Paper manufacturing
Printing products manufacturing	Petroleum and coal manufacturing	Chemical manufacturing
Plastics manufacturing	Wholesale trade	Retail trade
Transportation and warehousing	Information	Finance and insurance
Real estate and rental services	Professional and technical services	Mgmt. of companies and enterprises
Admin. and waste mgmt. services	Educational services	Health care and social assistance
Arts and entertainment services	Accommodation	Food services
Other services		

- Average share of 1.4% with standard deviation of 0.5% over time

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More about the estimation

Preferences

- Consumption shares β are taken directly from the data
- Relative risk aversion ρ is **estimated**

Production technique productivity shifters

- Function A_i as described earlier
- Set ideal shares α_{ij}° to their data average
- Costs κ_{ij} of deviating from α_{ij}° are **estimated**

Process for exogenous shocks ε_t

- Random walk with drift $\varepsilon_t = \gamma + \varepsilon_{t-1} + u_t^{\varepsilon}$, with $u_t^{\varepsilon} \sim$ iid $\mathcal{N}(0, \Sigma_t)$.
- Drift vec. γ and cov. mat. Σ_t are **backed out from the data given** (ρ, κ)**.**

Loss function: Target the full set of shares α_{ijt} and the GDP growth.

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More about the calibration

- Random walk with drift $\varepsilon_t = \gamma + \varepsilon_{t-1} + u_t$, with $u_t \sim$ iid $\mathcal{N}(0, \Sigma_t)$.
  - We estimate the vector γ by averaging $\Delta\varepsilon_t = \varepsilon_t - \varepsilon_{t-1}$ over time
  - We estimate Σ_t as

$$\hat{\Sigma}_{ijt} = \sum_{s=1}^{t-1} \lambda^{t-s-1} u_{is} u_{js}$$

    where $\hat{\lambda} = 0.47$ is set to the sectoral average of the corresponding parameters of a GARCH(1,1) model estimated on each sector's productivity innovation u_{it}

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Expression for $\zeta(\alpha_i)$

The function $\zeta\left(\alpha_i\right)$ is

$$\zeta\left(\alpha_i\right)=\left[\left(1-\sum_{j=1}^{n}\alpha_{ij}\right)^{1-\sum_{j=1}^{n}\alpha_{ij}}\prod_{j=1}^{n}\alpha_{ij}^{\alpha_{ij}}\right]^{-1}$$

This functional form allows for a simple expression for the unit cost K

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Microfoundation for ”one technique” restriction and cost minimization

Key restriction

Each firm/industry i can only adopt one production technique.

- Each industry $i \in \{1, \ldots, n\}$ has a continuum of firms $l \in [0, 1]$.
- Buyers use *shoppers* to purchase goods
  - Shoppers face an *information problem* and cannot differentiate between producers within an industry
  - Uniform allocation: each producer gets mass $Q_i dl$ of shoppers
  - Shoppers from firm m in industry j faces average price $\tilde{P}_i^{jm} = \int_0^1 \tilde{P}_{il}^{jm} dl$ for good i.
- When a shopper m from j meets a producer l from $i \to$ Nash bargaining

$$\tilde{P}_{il}^{jm} - K_i\left(\alpha_i^l, \left\{\tilde{P}_k^{il}\right\}_k\right) = \gamma\left(B_i^{jm} - K_i\left(\alpha_i^l, \left\{\tilde{P}_k^{il}\right\}_k\right)\right)$$

- Technique choice problem

$$\max_{\alpha_i^l \in \mathcal{A}_i} \mathrm{E}\left[\Lambda \sum_{j=0}^{n} Q_{ji} dl \int_0^1 \gamma\left(B_i^{jm} - K_i\left(\alpha_i^l, \left\{\tilde{P}_k^{il}\right\}_k\right)\right) dm\right] \longrightarrow \min_{\alpha_i^l \in \mathcal{A}_i} \mathrm{E}\left[\Lambda Q_i K_i\left(\alpha_i^l, \left\{\tilde{P}_k^{il}\right\}_k\right)\right]$$

Microfoundation for ”one technique” restriction and cost minimization

- Take limit $\gamma \to 0$
  - Nash bargaining implies $\tilde{P}^{jm}_{il} = K_i\left(\alpha^l_i, \left\{\tilde{P}^{il}_k\right\}_k\right) \to \tilde{P}^{jm}_{il}$ does not depend on $j, m \to \tilde{P}^{jm}_i \equiv P_i$.
  - $K_i\left(\alpha^l_i, \left\{\tilde{P}^{il}_k\right\}_k\right) \to K_i\left(\alpha^l_i, P\right)$
  - Cost minimization problem

$$\min_{\alpha^l_i \in \mathcal{A}_i} \mathrm{E}\left[\Lambda Q_i K_i\left(\alpha^l_i, \left\{\tilde{P}^{il}_k\right\}_k\right)\right] \longrightarrow \min_{\alpha^l_i \in \mathcal{A}_i} \mathrm{E}\left[\Lambda Q_i K_i\left(\alpha^l_i, P\right)\right]$$

  - We have the same pricing equation as in benchmark model with all firms in i choosing same technique

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Risk aversion and ρ

Given the log-normal nature of uncertainty $\rho \lesseqgtr 1$ determines whether the agent is risk-averse or not. To see this, note that when $\log C$ normally distributed, maximizing

$$\mathrm{E}\left[C^{1-\rho}\right]$$

amounts to maximizing

$$\mathrm{E}\left[\log C\right] - \frac{1}{2}\left(\rho - 1\right)\mathrm{V}\left[\log C\right].$$

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Impact of μ and Σ for α

Assumption (Weak complementarity)

For all $i \in \mathcal{N}$, the function a_i is such that $\frac{\partial^2 a_i(\alpha_i)}{\partial \alpha_{ij} \partial \alpha_{ik}} \geq 0$ for all $j \neq k$.

Lemma

Let $\alpha^* \in \text{int}(\mathcal{A})$ be the equilibrium network and suppose that the assumption holds. There exists a $\overline{\Sigma} > 0$ such that if $|\Sigma_{ij}| < \overline{\Sigma}$ for all i, j, there is a neighborhood around α^* in which

1. an increase in μ_j leads to an increase in the shares α^*_{kl} for all k, l;
2. an increase in Σ_{jj} leads to a decline in the shares α^*_{kl} for all k, l;
3. an increase in Σ_{ij} leads to a decline in the shares α^*_{kl} for all k, l.

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Pentagon example: parameter value

Details of the simulation:

1. *a* function: κ equal to 1, except $\kappa_{ii} = \infty$, α° are 1/10 except $\alpha^{\circ}_{ii} = 0$.
2. $\rho = 5$, $\beta = 0.2$. $\mu = 0.1$ except for $\mu_4 = 0.0571$. $\Sigma = 0.3 \times I_{n\times n}$ in Panel (a).
3. Panel (b): same as Panel (a) except $\mathrm{Corr}\,(\varepsilon_2, \varepsilon_4) = 1$.
4. Panel (c): same in Panel (a) except $\Sigma_{22} = 1$.

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We assume that $\kappa = \kappa^i \times \kappa^j$ where κ^i is an $n \times 1$ column vector and κ^j is an $1 \times (n+1)$ row vector.

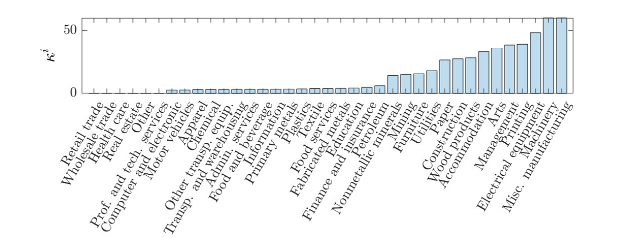
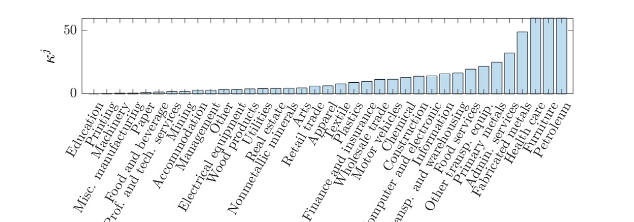


Figure 1: Vector of costs κ^i



Details of regressions

Volatility measures

- Supplier ΔVol_{t-1} is the 1-year lagged change in supplier-level volatility.
- Realized volatility is the 12-month standard deviation of daily stock returns from CRSP.
- Implied volatility is the 12-month average of daily (365-day horizon) implied volatility of at-the-money-forward call options from OptionMetrics.

Instrument

- As in Alfaro et al. 2019 "we address endogeneity concerns on firm-level volatility by instrumenting with industry-level (3SIC) non-directional exposure to 10 aggregate sources of uncertainty shocks. These include the lagged exposure to annual changes in expected volatility of energy, currencies, and 10-year treasuries (as proxied by at-the-money forward-looking implied volatilities of oil, 7 widely traded currencies, and TYVIX) and economic policy uncertainty from Baker et al 2016.. [...] To tease out the impact of 2nd moment uncertainty shocks from 1st moment aggregate shocks we also include as controls the lagged directional industry 3SIC exposure to changes in the price of each of the 10 aggregate instruments (i.e., 1st moment return shocks). These are labeled 1st moment 1st moment of IVs."

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